MODEL OF LIFETIME-EXTENDING MAINTENANCE[1]

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ABSTRACT
In this paper, a new maintenance model is presented in which both the interval of lifetime extension and the interval of preventive replacement can be optimised. Through lifetime extension, the deterioration can be delayed as such that failure is postponed and the lifetime of a component is extended. Through replacement, the condition of a component can be restored to its original condition. The lifetime-extending maintenance model (LEM model) can be used to optimise maintenance in both the design phase and the use phase. In the design phase, the initial cost of investment can be optimally balanced against the future cost of maintenance (life-cycle costing). In the use phase, the cost of preventive maintenance (lifetime extension and preventive replacement) can be optimally balanced against the cost of corrective maintenance (corrective replacement and failure). The cost-based criterion of the expected discounted costs over an unbounded time-horizon is used to compare different maintenance strategies. The LEM model enables optimal maintenance decisions to be determined on the basis of the uncertainties in the deterioration. It has been successfully applied in a case study to optimise the maintenance of a coating protecting steel.

INTRODUCTION
For safe and economic design of structures, there are many decision models available. Design optimisation has been a subject of interest for a long time. Although both safety and cost are influenced by time-related deterioration processes and maintenance actions, maintenance is not often considered in the design phase. Designers trying to build a good structure often take measures to avoid damages caused by these processes. Little attention is given to the relation between the investment in these measures, on the one hand, and the risk related to possible damage, on the other hand. Even maintenance managers pay little attention to the economic motivation of the actions taken. Common motives for maintenance are the experience and the personal opinion of the man in charge. Experience can be a good motive when the aim is to avoid failure. A more difficult task is finding an economic optimum for maintenance intervals on the basis of experience only. Therefore, maintenance optimisation is of growing interest and optimisation tools are needed. Such a tool is presented in this paper. It can be applied to perform life-cycle cost analyses for many different structures and many different situations. The development of maintenance management systems based on balancing cost and reliability using life-cycle costing is still in its infancy. Recently, life-cycle costing has been applied to the maintenance of the following structures: bridges (Frangopol, 1998), concrete structures in nuclear plants (Mori & Ellingwood, 1993), and hydraulic structures (Van Noortwijk, 1996).

[1] This work was published in M.C. Forde, editor, *Proceedings of the 8th International Conference on Structural Faults and Repair, London, United Kingdom, 13-15 July 1999*. Edinburgh: Engineering Technics Press, 1999.

MAINTENANCE DECISIONS MODELS

The Lifetime-Extending Maintenance model (LEM model) is aimed at minimising the cost of investment, lifetime-extending maintenance, and replacement. A structure can be expensive to build, but may require little maintenance to reach a long lifetime. Alternatively, one can save building cost and perform more frequent maintenance to reach the same lifetime. This long lifetime may not be necessary when the expected economic lifetime is dominant. The LEM model was built as a tool for optimising maintenance actions that are related to design decisions. An optimal combination of design and maintenance can thus be obtained. The result should be calibrated by frequent inspection of the structure, and the maintenance strategy should be evaluated and adjusted if necessary.

In this paper, the term "lifetime-extending maintenance" is defined as "activities that can be carried out to extend the lifetime of (a component of) a structure". Depending on the level of detail, a replacement on one level can be a lifetime-extending maintenance action on another level. A lifetime-extending maintenance action can either delay the deterioration or improve the current condition. An example of delaying the deterioration is the oiling of a machine. An example of improving the current condition is replacing the sealing of a cylinder (to extend the lifetime of that cylinder). The LEM model can be used to determine an optimal design and maintenance strategy for critical components that are often used under comparable circumstances (comparable with respect to both cost and deterioration). All parameters should be studied thoroughly.

REPLACEMENT AND LIFETIME-EXTENSION MODEL

The main difference between the LEM model and most other standard replacement models is the kind of physical relation which is simulated. In replacement models, a replacement interval is optimised with respect to the cost of preventive and corrective replacement. This can be considered as a one-dimensional decision problem with the preventive replacement interval being the decision variable. Such a model is called an age replacement model (see, e.g., Barlow & Proschan, 1996). The result is an optimal replacement interval for which the expected costs are minimal (Figure 1).

The LEM model can be considered as a two-dimensional decision problem with two decision variables: the preventive replacement interval and the lifetime-extension interval. The result is an optimal combination of these two intervals for which the expected costs are minimal (Figures 7 and 9).

When an optimisation is carried out, one has to figure out which simulation model is most suitable. The LEM model should only be used when the lifetime extension and the replacement interval are interrelated.

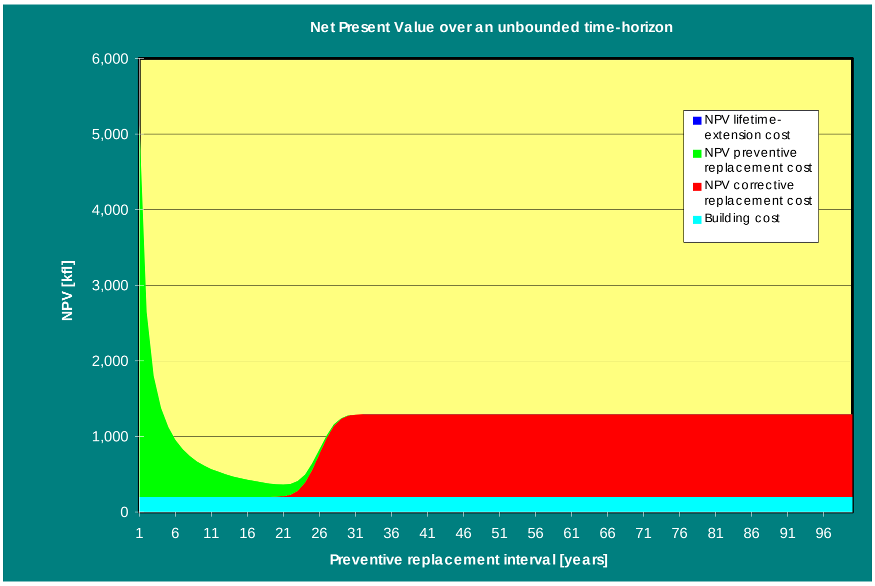

Figure 1. Output of the age replacement model.

BASIC CHARACTERISTICS OF THE LEM MODEL

The LEM model is based on a replacement model to which a LEM module has been added. A deterioration process of (a component of) a structure in a LEM simulation can usually be subdivided into two parts: (i) an initiation period and (ii) a propagation period. During the initiation period, the lifetime-extending measure is fully effective and the protected component does not deteriorate at all. During the propagation period, the lifetime-extending measure loses its effectiveness and the deterioration sets in. The "net propagation curve" (Figures 2-3) is defined as the curve describing the condition without lifetime extension as a function of time, starting at the beginning of the propagation period. Lifetime-extending measures can be superposed on this net propagation curve, which results in the overall condition with lifetime extension.

Possible lifetime-extending measures are:

- starting a new initiation period (Figure 2);
- improving the component's condition (Figures 2-3);
- changing the rate of deterioration:

- *Repeating:* After every lifetime extension the rate of deterioration is the same and equals the rate of deterioration of the net propagation curve at time zero; in other words, after every lifetime extension the propagation curve repeats (Figure 2);
- *Non-repeating:* After a lifetime extension the propagation curve is the same as the net propagation curve at the corresponding condition (Figure 3).

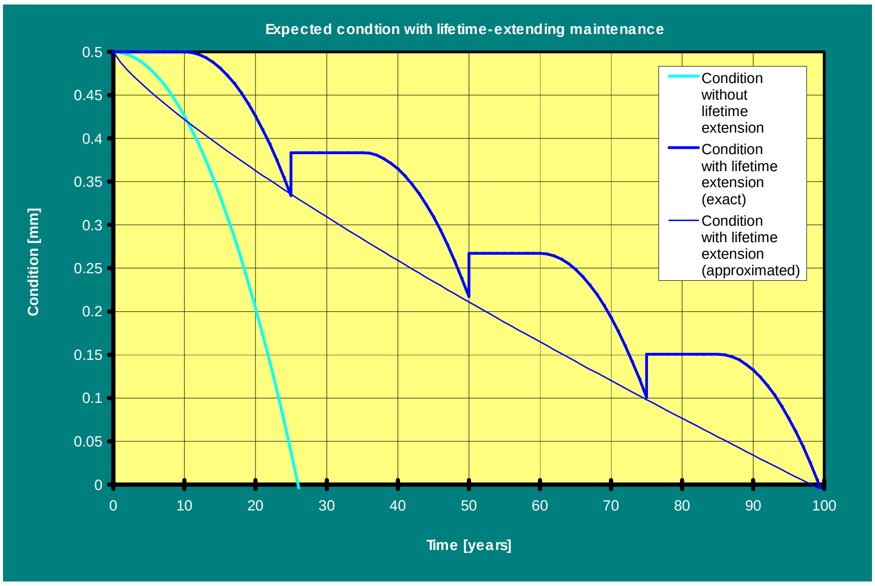

Figure 2. Example of lifetime-extending maintenance with repeating propagation curve.

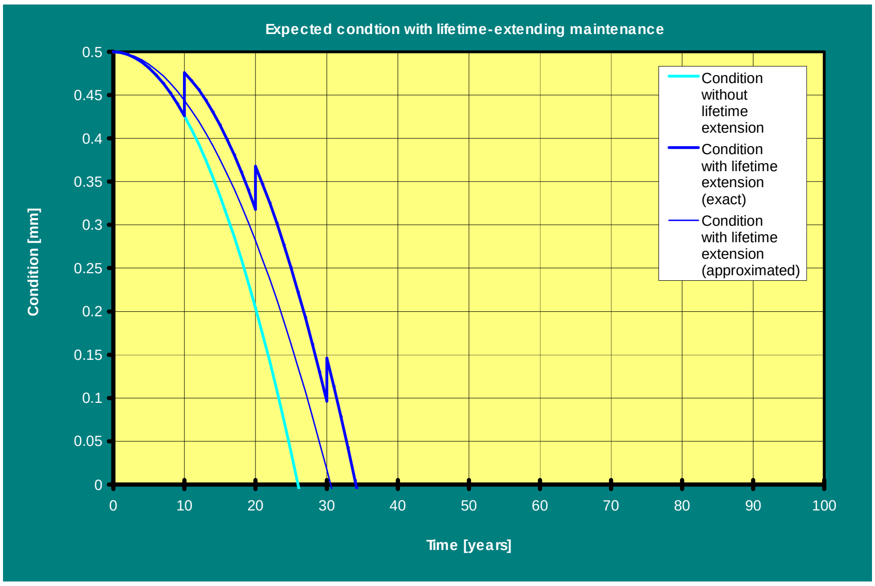

Figure 3. Example of lifetime-extending maintenance with non-repeating propagation curve.

For modelling purposes, the expected condition with lifetime extension has been approximated by the "resultant propagation curve" (Figure 2-3). Accordingly, the expected lifetime is approximately equal to the time at which the expected deterioration equals the failure level (in this example 0 mm). Depending on how accurate the deterioration can be predicted, the uncertainty in the deterioration can be specified in terms of the uncertainty distribution of the deterioration (for an example, see Figure 6).

For each combination of the interval of lifetime extension and the interval of preventive replacement, the LEM model calculates the expected discounted costs over an unbounded horizon. These expected discounted costs are determined by summing the present discounted values of the costs over an unbounded horizon. Since the future cost is discounted on the basis of a discount rate (usually defined as the nominal rate of interest minus the rate of inflation), it is possible to compare the value of money at different dates. According to Van Noortwijk (1998), these expected discounted costs over an unbounded horizon can be easily derived using te discrete renewal theorem. The expected discounted costs over an unbounded horizon are also denoted by the Net Present Value (NPV).

SIMULATION PARAMETERS

The input of the LEM model consists of deterioration parameters, lifetime-extension parameters, and cost parameters.

The deterioration parameters determine the net propagation curve from the time at which the deterioration starts until the time at which the condition equals the failure level (without taking lifetime-extending maintenance into account).

The lifetime-extension parameters describe the effect on the net propagation curve after carrying out lifetime-extending maintenance, as described above.

The four cost parameters that can be distinguished are: the cost of investment (building), the cost of preventive replacement (replacing before failure), the cost of corrective replacement (replacing after failure) and the cost of lifetime extension.

For each combination of the interval of lifetime extension and the interval of replacement, the Net Present Value is calculated. All results are gathered in a two-dimensional NPV matrix.

The output of the LEM model consists of the optimal lifetime-extension interval and the optimal replacement interval, as well as the corresponding expected time to failure and the minimal Net Present Value. Notice that if corrective replacement is optimal, the optimal preventive replacement interval is (theoretically) unbounded.

EXAMPLE: COATING OF A STEEL STRUCTURE

How the model works is illustrated by the following fictive example. A steel structure consisting of slabs with welded joints is protected by a coating. The coating must be maintained in order to perform its function. There is a relation between the maintenance of the coating (Component 2), on the one hand, and the lifetime of the steel (Component 1), on the other hand. In this paper, lifetime-extending maintenance is defined to be maintenance of one component (e.g. the coating) to extend the lifetime of another component (e.g. the steel). Notice that this example is not based on real deterioration data of coatings protecting steel. Therefore, the values of the parameters may not be completely correct.

The five steps of LEM modelling are described next.

<u>Step 1. Inventorying the main deterioration processes that influence the two components</u>

Before the modelling is started one should investigate the deterioration processes of the steel and the coating, and specify how they influence each other:

A. Deterioration influencing the steel:
   - I) corrosion at the welded joints of the structure (critical parts);
   - II) corrosion of the slabs of the structure;
   - III) fatigue.

B. Deterioration influencing the coating:
   - I) ageing of the coating at joints and angles
     (actually, the different ageing processes should be evaluated);
   - II) ageing of the coating at the slabs;
   - III) local damaging of the coating.

<u>Step 2. Defining a one-to-one relation between the deterioration processes of the structure</u>

A one-to-one relation between the deterioration processes of two components exists, if it can be isolated from the other deterioration processes. In other words, the deterioration processes should be (more or less) independent.

In this situation, deterioration process A.III can be neglected because it has little influence on the overall deterioration. Process A.III is more or less independent of the other processes, and can be considered separately.

Process A.I is related to B.I, and process A.II is related to B.II. The relation between A.I and B.I is considered to be independent of the relation between A.II and B.II, since there is no direct interaction between the ageing of both areas. Although maintenance of the coating at the joints is usually considered to extend the lifetime of the coating of the structure, this is not strictly correct. The coating of the slabs does not benefit from the maintenance of the joints.

<u>Step 3. Setting up the simulation of lifetime extension</u>

Once the main deterioration processes and their critical one-to-one relations are found, the correct simulation model must be defined. In order to set up the right simulation, the following five questions must be answered:

Question	Answer
a) What component should be preserved by lifetime-extending maintenance?	The steel.
b) What is the primary deterioration process?	Loss of steel thickness due to corrosion.
c) What is the condition parameter?	The surplus in average steel thickness for which corrosion is allowed.
d) What is failure?	Average steel thickness is less than required according to the building code.
e) What lifetime-extending maintenance action can be taken?	Applying a coating

For the welded joints (critical areas) and the slabs, separate simulations have been performed.

<u>Step 4. Specifying the model parameters</u>
Since the simulation is now defined, the model parameters should be specified. These parameters have been assessed using informal expert judgement.

Deterioration parameters
In this simulation, the steel is assumed to have a surplus steel thickness to allow for corrosion of 0.5 mm, for both the slabs and the joints. After losing this surplus thickness, the structure does not have the safety which is required in the building code (this does not mean that the structure collapses). The average steel thickness is regarded as the condition parameter, where it is assumed that the "good parts" compensate for the "bad parts". The variation between the good and bad parts can be incorporated into the uncertainty distribution of the average steel thickness.
At the end of the initiation period, the coating begins to protect insufficiently and the first corrosion occurs. The effectiveness of the coating at the corroded area decreases in time and the steel will finally corrode as if there were no coating (Figure 4). This so-called rate of free corrosion is then considered to be constant.

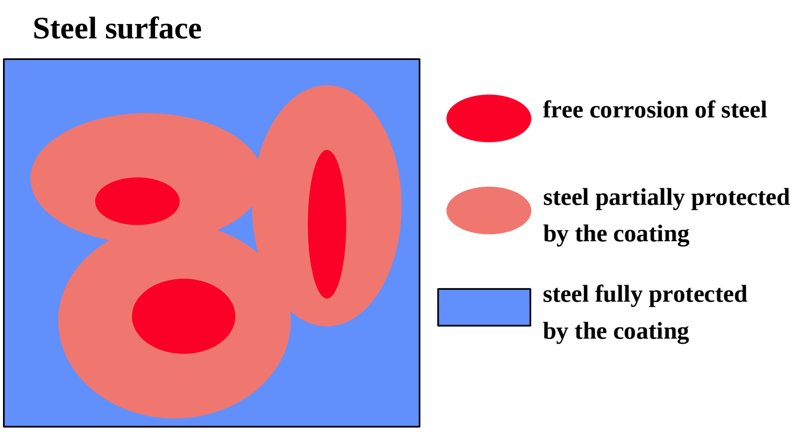

Figure 4. Corrosion at the slabs.

The first corrosion at the joints starts earlier than at the slabs. Moreover, the variation in the time to first corrosion at the joints is larger than the variation in the time to first corrosion at the slabs. The end of the initiation period is defined to be the time at which the first corrosion occurs at any point of that area (the lower bound of the 90 percent confidence interval could be used). The initiation period is succeeded by the propagation period. At the beginning of the propagation period, the average rate of corrosion equals zero. In time, the average rate of corrosion approaches the rate of free corrosion (the howl surface is corroding; the upper bound of the 90 percent confidence interval could be used). Between these two extremes, the average rate of corrosion changes in time. In the LEM model this "net propagation curve" is approximated by the power law $y(t) = c - at^b$, where $y(t)$ represents the condition at time t. The power b is about 2, since the corroded areas are assumed to grow as a quadratic function of time. On the basis of the initiation period, the expected time to free corrosion, the initial condition, and the rate of free corrosion, the coefficients a and c can be determined. The expected time to failure can be approximated by solving the equation $y(t) = 0$ for t (Figure 5).

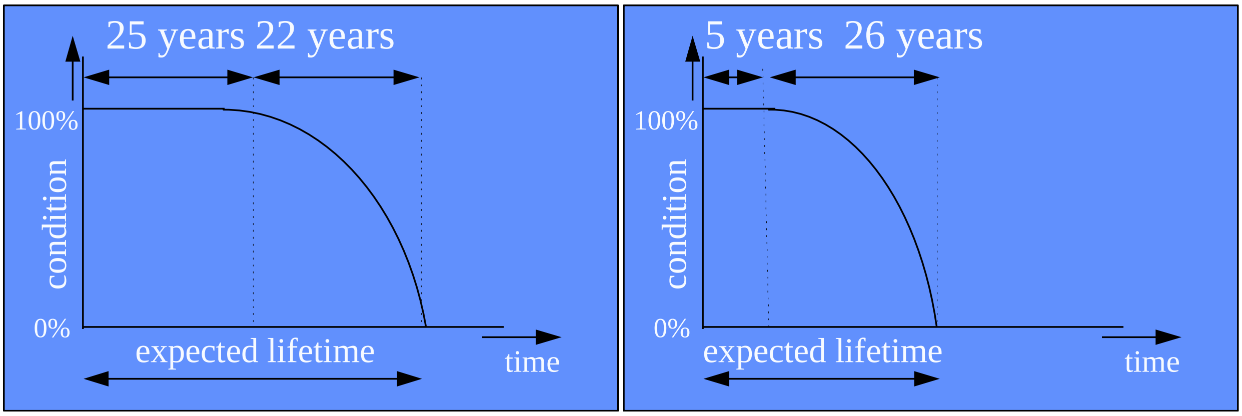

Figure 5. Expected deterioration without lifetime extension for the slabs (left) and the joints (right).

The uncertainty in the expected deterioration has been modelled using a so-called gamma process. For details on the gamma process with expected deterioration being linear and non-linear in time, see Van Noortwijk (1998) and Van Noortwijk & Klatter (1999), respectively. In Figure 6, the expected deterioration of the slabs is displayed for the situation without lifetime extension, together with its 5th and 95th percentile. On the basis of the stochastic process of deterioration, the probabilities of failure can be easily obtained.

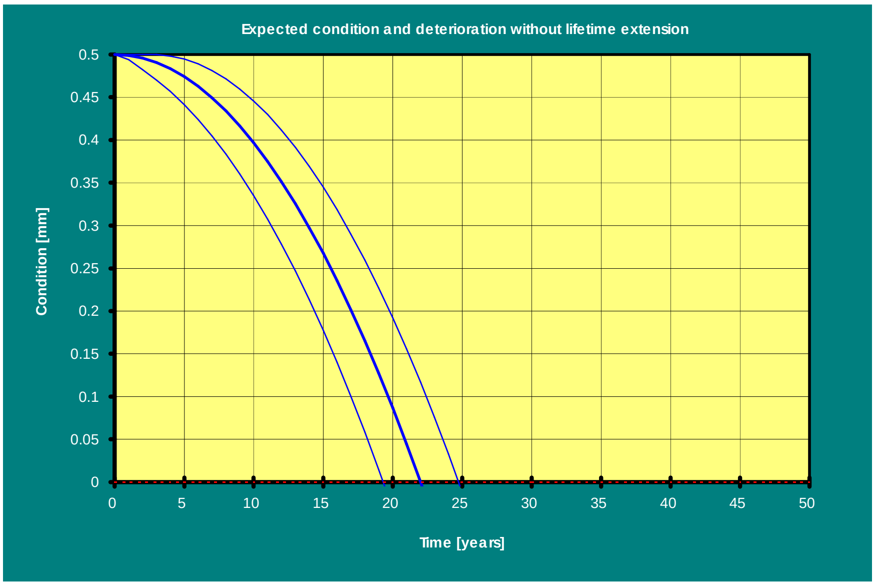

Figure 6. Expected conditon without lifetime extension for the slabs including the 5th and 95th percentile.

Deterioration parameters LEM model	Slabs		Joints	
Initial condition	0.5	mm (average thickness/area)	0.5	mm (average thickness/length)
Failure level	0	mm (average)	0	mm (average)
Length initiation period (LEM parameter)	25	years	5	years
Expected time to free corrosion	50	years	40	years
Rate of free corrosion	0.05	mm/year	0.05	mm/year
$y(t) = c - at^b$, $c =$	0.5		0.5	
$a =$	0.001		0.001	
Expected time to failure	47	years	31	years
Uncertainty (coefficient of variation)	15	%	15	%

Lifetime-extension parameters
For both the slabs and the joints, Figure 5 shows the expected condition of the steel as a function of time when no lifetime-extending maintenance is performed. The expected lifetime can be extended by mechanically removing the old coating and corrosion. Unfortunately, some steel will be removed as well. A new coating can be placed, thus starting a new initiation period. After the initiation period, the propagation curve has the same shape as the original curve (repeating propagation curve).

Lifetime-extension parameters LEM model	Slabs		Joints	
Length initiation period	25	years	5	years
Condition improvement	-0.1	mm	-0.1	mm
Repeating		yes		yes

Cost parameters
The costs of maintenance can roughly be subdivided into four types:

- *Cost of initial investment:* Cost of building a new structure;
- *Cost of preventive replacement:* Cost of replacing an existing structure by a new one before failure;
- *Cost of corrective replacement:* Cost of replacing an existing structure by a new one after failure plus the cost of failure. In this example the cost of failure is zero, because failure is defined as the event in which the structure does not satisfy the prescribed safety norm which is defined in the building codes (no physical failure).
- *Cost of lifetime-extending maintenance:* Cost of extending the lifetime of a structure.

In addition to the above cost parameters, a discount rate must be assessed. By calculating the Net Present Value, the initial cost of investment can be optimally balanced against the future cost of maintenance. The expected costs are calculated with respect to an unbounded time-horizon, assuming a structure to be replaced directly after either failure or preventive replacement.

Furthermore, attention must be given not to count certain cost twice. In this case study, lifetime-extending maintenance of the slabs can be combined with lifetime-extending maintenance of the joints. To achieve this, the cost of lifetime-extending maintenance at the joints should be subtracted from the cost of lifetime-extending maintenance at the slabs. Since the slabs and joints are not replaced separately, the cost of preventive and corrective replacement of either the joints or the slabs should be left out of consideration. This is only permitted if the expected lifetime of the component that is not considered is significantly larger than the expected lifetime of the other component (the probability that the lifetime of the component with the largest expected lifetime is smaller than the lifetime of the other component is hereby neglected). However, neglecting the cost of failure is not realistic. Therefore, it is suggested to perform the simulation twice: once separately (assuming the cost of preventive and corrective replacement to be equal to the actual cost) and once combined (assuming the cost of preventive and corrective replacement to be zero for the slabs) in order to get some idea of the neglected risk.

Cost parameters LEM model	**Slabs**		**Joints**	
cost of preventive replacement (separately)	2000	× fl 1000	2000	× fl 1000
cost of preventive replacement (combined)	0	× fl 1000	2000	× fl 1000
cost of corrective replacement (separately)	2000	× fl 1000	2000	× fl 1000
cost of corrective replacement (combined)	0	× fl 1000	2000	× fl 1000
cost of lifetime-extending maintenance	200	× fl 1000	100	× fl 1000
discount rate	4	%	4	%

Step 5. Performing simulations with the LEM model

Since all input parameters are specified, simulations can be performed with the LEM model. The program provides two options:

- *Single simulation:* A simulation when a single interval of lifetime-extending maintenance is given; this results in an optimal preventive replacement interval with minimal Net Present Value.
- *Matrix simulation:* The Net Present Value is calculated for each combination of the preventive replacement interval and the lifetime-extension interval; this results in an optimal combination of preventive replacement and lifetime extension with minimal Net Present Value.

The LEM model has been used to obtain optimal lifetime-extending maintenance decisions for both the slabs and the joints.

Lifetime extension of the slabs

For the slabs the optimal interval of lifetime-extending maintenance (in terms of replacing the coating) is 32 years combined with corrective replacement. A three-dimensional plot of the Net Present Value as a function of the lifetime-extension interval and the replacement interval is shown in Figure 7. The corresponding expected condition of the steel at the slabs is shown in Figure 8; the expected lifetime under optimal lifetime extension is 106 years. Including the building cost the minimal NPV under lifetime extension equals fl 2,126,000.

Lifetime extension of the joints

For the joints the optimal interval of lifetime-extending maintenance (in terms of replacing the coating) is 14 years combined with corrective replacement. A three-dimensional plot of the Net Present Value as a function of the lifetime-extension interval and the replacement interval is shown in Figure 9. The corresponding expected condition of the steel at the joints is shown in Figure 10; the expected lifetime under optimal lifetime extension is 44 years. Including the building cost the minimal NPV under lifetime extension equals fl 2,561,000. Both longer and shorter lifetime-extension intervals result in a shorter expected lifetime and a higher NPV. In Figure 11, a part of the NPV matrix is given for the joints.

Lifetime extension of the slabs and joints combined

If replacement of the coating at the slabs and the joints is combined, a proper lifetime-extending maintenance strategy might be a lifetime-extension interval for the slabs and the joints of 28 and 14 years, respectively. Since the expected lifetime of the slabs is larger (106 years) than the expected lifetime of the joints (44 years), this combined lifetime-extension strategy mainly results in corrective replacement of the joints and preventive replacement of the slabs (where the latter is determined by the former). Using the NPV matrix of the LEM model, the corresponding NPV is approximately equal to fl 2,642,333.

In this example, more maintenance does not lead to a longer lifetime, since each time maintenance is performed some steel diameter is lost (the exact amount is not known). Hence, the case study was also studied for less loss of steel diameter. This leads to more realistic lifetimes. Furthermore, the initial condition was varied. If there is little extra steel thickness, short lifetimes are realised. Therefore, an overlayment strategy was simulated, in which there was little corrosion allowed and the coating would have to be removed after four times overlaying. This part of the case study is not included in this paper.

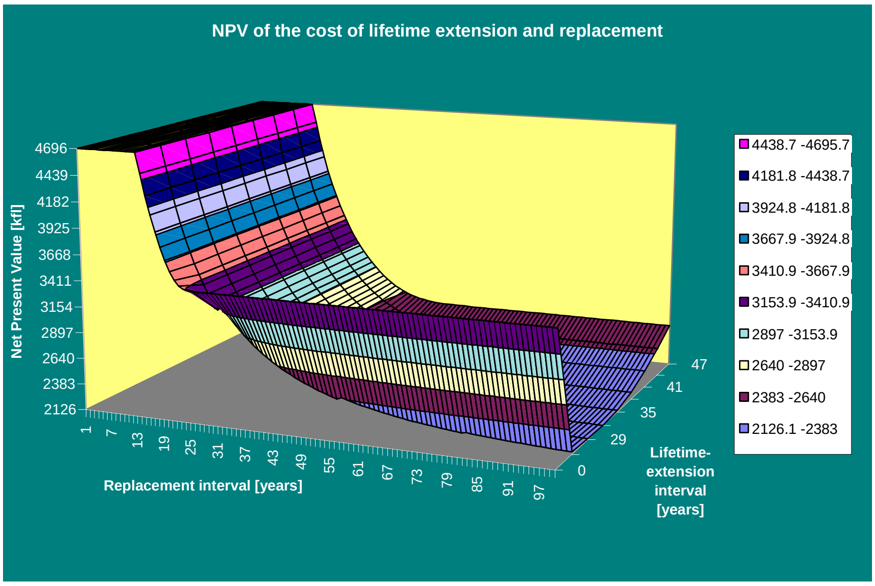

Figure 7. NPV of the expected cost of lifetime extension and replacement for the slabs.

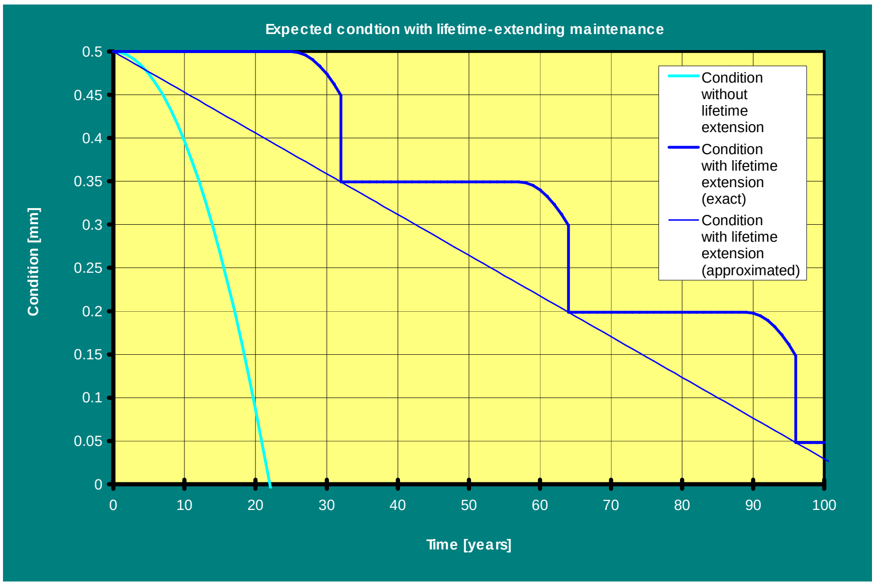

Figure 8. Expected condition with and without lifetime extension for the slabs.

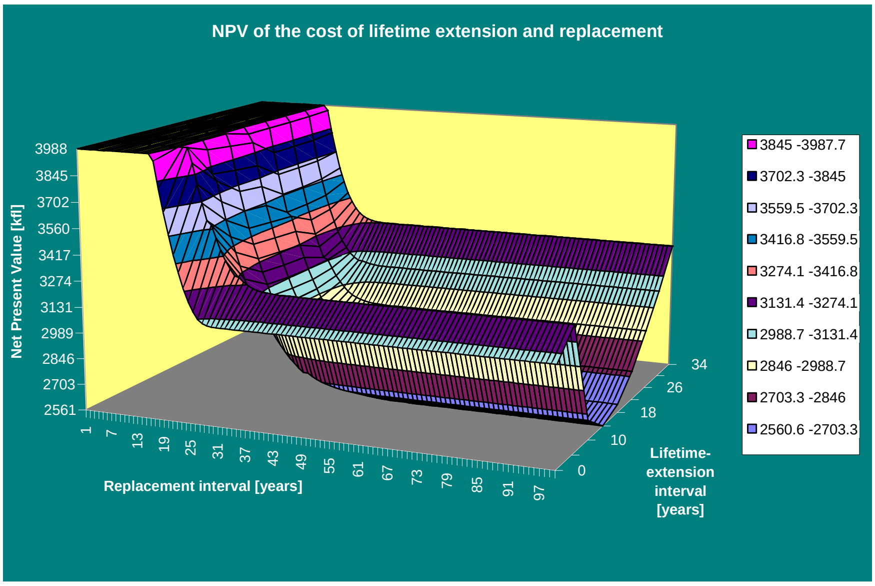

Figure 9. NPV of the expected cost of lifetime extension and replacement for the joints.

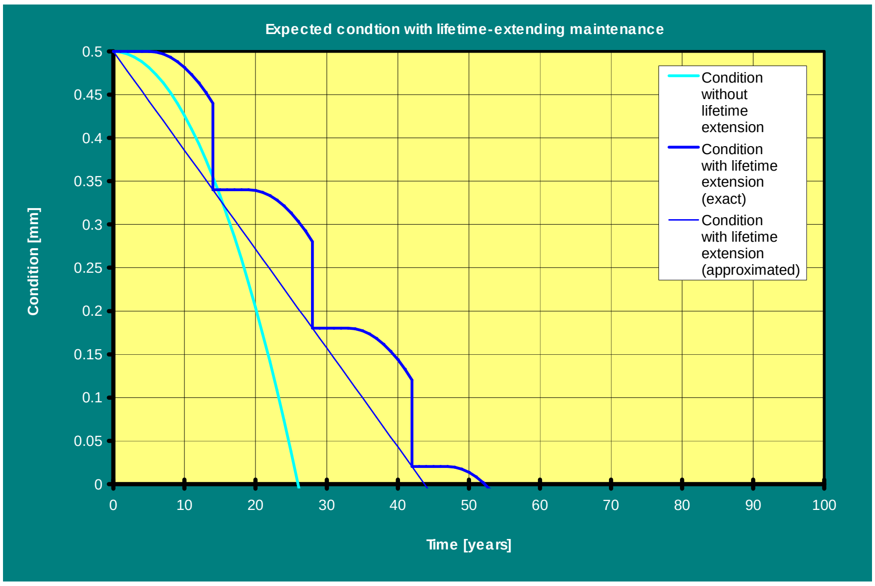

Figure 10. Expected condition with and without lifetime extension for the joints.

replacement-interval [years]	LEM interval [years]								
	0	6	10	14	18	22	26	30	34
41	3094,64	3232,85	2745,65	2647,76	2659,2	2733,23	2890,77	3061,959	3274,13
42	3094,64	3232,5	2729,72	2630,22	2647,31	2729,12	2890,25	3061,94	3274,13
43	3094,64	3232,46	2716,17	2630,04	2637,58	2726,19	2889,95	3061,931	3274,13
44	3094,64	3232,35	2704,78	2616,83	2629,74	2724,14	2889,78	3061,928	3274,13
45	3094,64	3232,3	2695,35	2605,59	2623,54	2724,08	2889,69	3061,926	3274,13
46	3094,64	3232,27	2687,64	2596,15	2618,7	2723,15	2889,64	3061,926	3274,13
47	3094,64	3232,26	2681,44	2588,32	2615,01	2722,55	2889,62	3061,925	3274,13
48	3094,64	3232,25	2676,52	2581,91	2612,23	2722,17	2889,61	3061,925	3274,13
49	3094,64	3232,25	2672,68	2576,75	2610,17	2721,93	2889,6	3061,925	3274,13
50	3094,64	3232,25	2669,73	2572,63	2608,68	2721,79	2889,6	3061,925	3274,13
51	3094,64	3232,25	2669,67	2569,41	2607,62	2721,7	2889,6	3061,925	3274,13
52	3094,64	3232,25	2668,01	2566,92	2606,88	2721,65	2889,6	3061,925	3274,13
53	3094,64	3232,25	2666,79	2565,02	2606,37	2721,62	2889,6	3061,925	3274,13
54	3094,64	3232,25	2665,92	2563,6	2606,03	2721,61	2889,6	3061,925	3274,13
55	3094,64	3232,25	2665,3	2562,55	2606,02	2721,6	2889,6	3061,925	3274,13
56	3094,64	3232,25	2664,86	2561,79	2605,88	2721,6	2889,6	3061,925	3274,13
57	3094,64	3232,25	2664,57	2561,79	2605,78	2721,59	2889,6	3061,925	3274,13
58	3094,64	3232,25	2664,37	2561,41	2605,72	2721,59	2889,6	3061,925	3274,13
59	3094,64	3232,25	2664,23	2561,14	2605,68	2721,59	2889,6	3061,925	3274,13
60	3094,64	3232,25	2664,15	2560,95	2605,66	2721,59	2889,6	3061,925	3274,13
61	3094,64	3232,25	2664,14	2560,83	2605,65	2721,59	2889,6	3061,925	3274,13
62	3094,64	3232,25	2664,11	2560,75	2605,64	2721,59	2889,6	3061,925	3274,13
63	3094,64	3232,25	2664,09	2560,69	2605,64	2721,59	2889,6	3061,925	3274,13

Figure 11. Part of the NPV matrix for the joints for combinations of lifetime-extension interval and preventive replacement interval.

CONCLUSIONS

A new lifetime-extending maintenance model (LEM model) was presented with which the intervals of lifetime extension and replacement can be optimised in the design and use phase. In the design phase, the initial cost of investment can be optimally balanced against the future cost of maintenance: i.e. the larger a structure’s initial condition, the higher the cost of investment, but the lower the cost of maintenance. In the use phase, the cost of preventive maintenance (lifetime extension and preventive replacement) can be optimally balanced against the cost of corrective maintenance (corrective replacement and failure): i.e. the more preventive maintenance is carried out, the higher the cost of preventive maintenance, but the smaller the cost of corrective maintenance. The cost-based criterion of the expected discounted costs over an unbounded time-horizon (Net Present Value) is used to compare different maintenance strategies. The LEM model enables optimal maintenance decisions to be determined on the basis of the uncertainties in the deterioration. It has been successfully applied to optimise the maintenance of a coating protecting steel.

The LEM model has been implemented as an Excel spreadsheet. It can be obtained by sending an e-mail to either “j.d.bakker@bwd.rws.minvenw.nl” or “j.m.van.noortwijk@hkv.nl”.

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