Probability of dike failure due to uplifting and piping

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ABSTRACT: A methodology is presented for determining the probability of dikes failing due to uplifting and piping using directional sampling. The study concerns one dike section in the lower river area of The Netherlands. For this dike section, the three most important random quantities are: the North-Sea water level, the river Rhine discharge, and the critical head in the event of uplifting and piping. Dike failure due to uplifting and piping is defined as the event in which the resistance (the critical head) drops below the stress (the outer water level, a combination of both sea level and river discharge, minus the inner water level). Since the critical head is correlated over the length of a dike, the spatial variation and correlation is modelled using a Markovian dependency structure. Three-dimensional directional sampling on the basis of the polar coordinates of the sea water level, the river discharge, and the critical head is used to determine the failure probability.

1 INTRODUCTION

The aim of the paper is determining the probability of dikes failing due to uplifting and piping. Uplifting occurs when the covering layer of a dike bursts due to the high water pressure, whereas piping under dikes occurs due to the entrainment of soil particles by the erosive action of seepage flow.

This study concerns one dike section of the dike ring Hoeksche Waard (dike section 13.1), which is situated in the lower river area of The Netherlands. The three most important random quantities representing inherent uncertainties are: the North-Sea water level, the river Rhine discharge, and the critical head in the event of uplifting and piping. In this situation, dike failure due to uplifting and piping is defined as the event in which the resistance (the critical head) drops below the stress (the outer water level, a combination of both sea level and river discharge, minus the inner water level). The statistical uncertainties represent the uncertainties in the parameters of the probability distributions of the sea water level and the river discharge.

Since the critical head is correlated over the length of a dike section, the spatial variation and correlation of the critical head in the event of uplifting and piping is modelled using a Markovian dependency structure. This means that the random quantities representing the inherent uncertainties in the critical head of one dike subsection only depend on the values of the corresponding random quantities in the two adjacent dike subsections.

The probabilities of failure due to uplifting and piping are calculated in three steps.

First, a dike section is subdivided into smaller subsections by assuming the limit-state function of uplifting and piping to be a Gaussian stationary process and using the theory of the level-crossing problem.

Second, the failure probabilities of one dike subsection and two adjacent dike subsections are calculated using directional simulation.

Third, the failure probability of one dike section is determined by approximating the Gaussian stationary process by a Markov process with respect to failure of dike subsections.

Three-dimensional directional sampling is used to determine the probability of failure due to uplifting and piping. An advantage of three-dimensional directional sampling is that large sample sizes are not required. The results of directional sampling are compared to the results of First Order Reliability Method (FORM).

The outline of the paper is as follows. The limit-state functions of uplifting and piping are introduced in Section 2. The modelling of the spatial correlation and variation of the critical head for uplifting and piping, as well as determining the failure probability of a dike ring (series system of dike subsections) is studied in Section 3. The directional sampling technique, which is used to obtain the probability of failure due to uplifting and piping, is presented in Section 4. Results and conclusions can be found in the last section.

2 FAILURE DUE TO UPLIFTING AND PIPING

Uplifting occurs when the covering layer of a dike bursts, due to the high water pressure, whereas piping under dikes occurs due to the entrainment of soil particles by the erosive action of seepage flow. The limit-state function of uplifting and piping is given by (see also Figure 1)

$$Z = H_{up} - M_h(H + M_z - H_b) \qquad (1)$$

and

$$H_{up} = \max\{M_u H_u, M_p H_p\} \qquad (2)$$

with

H = outer water level [m +NAP],
H_b = inner water level [m +NAP],
H_p = critical head in the event of piping [m],
H_u = critical head in the event of uplifting [m],
H_{up} = critical head for uplifting and piping [m],
M_h = model factor water level [-],
M_p = model factor piping [-],
M_u = model factor uplifting [-],
M_z = model factor water-flow model ZWENDL [m].

The two critical heads H_u and H_p are functions of the following random quantities: the volume weights of sand and water; the angular rolling friction; the constant of White; the sizes of sand particles; the permeability; the thickness of the covering layer and the sand layer; and the dike width. These random quantities are independent, lognormally distributed, and represent inherent uncertainties in the critical head and are correlated over the length of the dike on the basis of the quadratic exponential correlation function in Eq. (3).

The probability distribution of the outer water level H is a mixture of both the probability distribution of the North-Sea water level at Hoek of Holland, denoted by S [m +NAP], and the probability distribution of the river Rhine discharge at Lobith, denoted by Q [m^3/s]. The further down the river, the more the sea water level S affects the local water level H, and the less the river discharge Q affects the local water level H. Given a particular sea water level and a particular river discharge, the downstream water level can be obtained with the one-dimensional water-flow model ZWENDL. On the basis of ZWENDL calculations, the local water level H has been approximated by a bilinear function of the sea water level S and the river discharge Q.

The probability distribution of the annual maximum sea water level is a generalized Pareto distribution, whereas the probability distribution of the annual maximum river discharge is a piecewise exponential distribution. Besides the inherent uncertainties, the uncertainties in the statistical parameters of these two probability distributions are taken into acount.

The subject of study is the probability of dike failure due to uplifting and piping per two days. Therefore, the probability distributions of annual maximal sea level and discharge must be transformed to probability distributions of maxima over two days. These transformed random quantities are denoted by S_{2days} and Q_{2days}, respectively, and they are independent.

The other random quantities are distributed as follows. The inner water level H_b and the model factor of the water-flow model M_z have a normal distribution; the model factor for uplifting M_u and the model factor for piping M_p have a lognormal distribution; and the model factor of the water level M_h has a beta distribution. The two critical heads for uplifting and piping, the outer and inner water level, and the four model factors are mutually independent.

For further details about the probability distribution representing inherent and statistical uncertainties, we refer to Cooke & Van Noortwijk (1998).

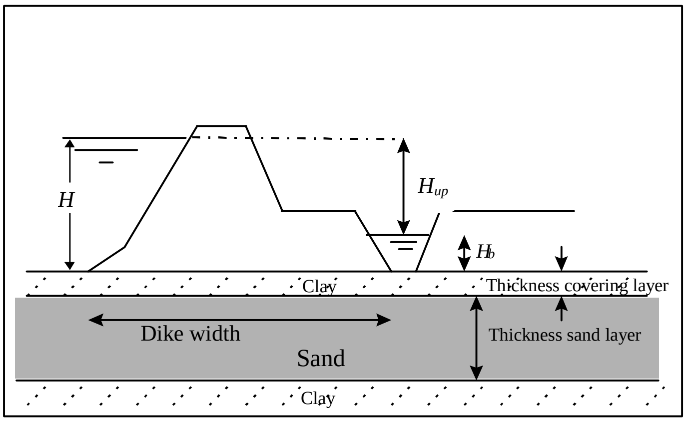

Figure 1. Cross-section of a dike.

3 SPATIAL CORRELATION AND VARIATION

In order to model the spatial variation and correlation of the random quantities representing the inherent uncertainties in the critical head, Vrouwenvelder (1993, Chapter 2) used the quadratic exponential correlation function

$$\rho(x) = \exp\left\{-\left[\frac{x}{d_x}\right]^2\right\}, \qquad (3)$$

where $\rho(x)$ is the product moment correlation, x is the distance between the two points at which the stochastic process is studied [m], and d_x is the fluctuation scale [m]. The fluctuation scale represents the spatial variation: the larger the fluctuation scale, the less spatial variation, and the larger the correlation coefficient. The parameters used to model the spatial variation can be found in Cooke & Van Noortwijk (1998).

The failure probability of a dike section can be approximated by regarding a dike section as a (series) system of smaller dike subsections. Dependencies between failure events of the dike subsections can then be modelled on the basis of a Markovian dependency structure. For modelling the spatial variation, this means that the random quantities in one dike subsection only depend on the values of the corresponding random quantities in the two adjacent dike subsections. The question arises how a dike section can best be subdivided into smaller subsections of length x^* and which value of the fluctuation scale d_x should be chosen. In this subsection, we present a methodology to obtain both the dike subsection length and the fluctuation scale.

On the one hand, we assume the limit-state function of uplifting and piping to be a Gaussian stationary process and we use the theory of the so-called level-crossing problem [see, e.g., Vrouwenvelder (1993), Papoulis (1965, Chapter 14), and Karlin & Taylor (1975, Chapter 9)]. On the other hand, we approximate this Gaussian stationary process by a Markov process.

The level-crossing problem reads as follows. Let a Gaussian stationary process $Z(x)$ be given with mean 0, standard deviation 1, and correlation function $\rho(x)$, where x is the distance between the two points at which the stochastic process is studied [m]. The correlation function must satisfy $\rho''(0) < \infty$. An upper bound for the probability of exceeding the level β in a dike subsection of x metres can be written as

$$p(x) = \frac{x}{2\pi} \exp\left\{-\frac{\beta^2}{2}\right\} \sqrt{-\rho''(0)}. \tag{4}$$

The smaller x, the better this upper bound can be used as an approximation. For the quadratic exponential correlation function, Eq. (3) transforms into

$$p(x) = \frac{\sqrt{2}}{2\pi} \exp\left\{-\frac{\beta^2}{2}\right\} \frac{x}{d_x}. \tag{5}$$

According to Vrouwenvelder (1993), we can choose the length of the dike subsection, denoted by x^*, as such that the corresponding probability of exceedence $p(x^*)$ equals $\Phi(-\beta)$:

$$p(x^*) = \Phi(-\beta) \approx \frac{1}{\beta\sqrt{2\pi}} \exp\left\{-\frac{\beta^2}{2}\right\} \tag{6}$$

or, similarly,

$$x^* = \frac{d_x \sqrt{\pi}}{\beta}. \tag{7}$$

The approximation for $\Phi(-\beta)$ in Eq. (6) can only be applied when $\beta > 2$. For example, by using directional simulation, the β for one subsection of dike section 13.1 has the value (see Section 4)

$$\beta = -\Phi^{-1}(1.65\,10^{-6}) = 4.65. \tag{8}$$

On the basis of the quadratic exponential correlation functions of the random quantities representing the inherent uncertainties in the critical head, the fluctuation scale d_x of the limit-state function or process $Z(x)$ remains to be determined.

A dike subsection is assumed to have a length of x^* metres. Standard Monte Carlo simulation can then be used to calculate the correlation coefficient of $Z(x^*)$ and $Z(2x^*)$, denoted by $\rho(x^*)$. According to the quadratic exponential correlation function, the correlation coefficient $\rho(x^*)$ equals

$$\rho(x^*) = \exp\left\{-\left[\frac{x^*}{d_x}\right]^2\right\} \tag{9}$$

or

$$d_x = \frac{x^*}{\sqrt{-\ln(\rho(x^*))}}. \tag{10}$$

Using $\beta = 4.65$, substitution of (7) into (9) results in

$$\rho(x^*) = \exp\left\{-\frac{\pi}{\beta^2}\right\} = 0.86. \tag{11}$$

The subsection length x^* satisfying Eq. (11) can be determined with the aid of the following Picard iteration process:

$$x^*_{n+1} = \frac{x^*_n \sqrt{\pi}}{\beta\sqrt{-\ln(\rho(x^*_n))}}, \quad n = 1,2,3,..., \tag{12}$$

where x^*_1 is the initial estimate.

The subsection length x^* can now be determined in the following seven steps:

First, we calculate the probability of failure due to uplifting and piping of the first dike subsection using directional simulation (see Section 4) and substitute the resulting β into Eq. (12).

Second, we sample from the probability distributions of the outer water level (a bilinear function of the sea water level and the river discharge), the inner water level, and the four model factors using standard Monte Carlo sampling. These random quantities do not depend on the dike subsection studied.

Third, we make an initial estimate of the subsection length x^*.

Fourth, we sample from the (lognormal) distributions representing the inherent uncertainties in the critical head of the *first* dike subsection and we calculate the corresponding samples of the critical head for uplifting and piping denoted by H_{up1}.

Fifth, we sample from the conditional (lognormal) distributions representing the inherent uncertainties in the critical head of the *second* dike subsection *given* the corresponding samples of the *first* dike subsection and we calculate the corresponding values of the critical head for uplifting and piping denoted by H_{up2}. Recall that for all inherent uncertainties in the critical head, the correlation between the first and second dike subsection is defined by a quadratic exponential correlation function at the value x^*.

Sixth, we estimate the sample correlation coefficient $\rho(x^*)$ on the basis of the Monte Carlo samples of $Z(x^*)$ and $Z(2x^*)$, and substitute it into Eq. (12).

Seventh, we determine a new estimate of the subsection length x^* using Eq. (12).

As long as the $(n+1)$-th estimate of x^* differs from the n-th estimate, we repeat steps 5-7. For dike section 13.1, the Picard iteration process results in subsection length $x^* = 50$ metres. For convenience, the subsection length is rounded off to units of 5 metres.

Probability plots of the Monte Carlo samples of H_{up1} and H_{up2} give evidence of lognormally distributed critical heads. Therefore, lognormal distributions can be fitted to the critical heads for the first and second dike subsection. Also the critical head for the combination of the first and second dike subsection, denoted by

$$H_{up12} = \min\{H_{up1}, H_{up2}\}, \qquad (13)$$

appears to be lognormally distributed. For further details about the parameters of these lognormal distributions, we refer to Cooke & Van Noortwijk (1998).

Given the subsection length, the probabilities of failure due to uplifting and piping must be calculated both for one dike subsection separately and for two adjacent dike subsections combined. On the basis of these two probabilities, the probability of failure for a (series) system of dike subsections can be approximated by regarding the failure of dike subsections as a Markov process. In mathematical terms, the probability of failure of a dike section of length l, denoted by $p(l)$, can now be written as a function of the failure probability of one subsection $p(x^*)$ and the failure probability of two adjacent subsections $p(2x^*)$ in the following manner:

$$\begin{aligned} p(l) &= 1 - \Pr\{\text{no failure } \#1\} \times \\ &\times [\Pr\{\text{no failure } \#2 \mid \text{no failure } \#1\}]^{(l/x^*)-1} = \qquad (14) \\ &= 1 - \frac{[1-p(2x^*)]^{(l/x^*)-1}}{[1-p(x^*)]^{(l/x^*)-2}}, \end{aligned}$$

where $\Pr\{\text{failure } \#i\}$ denotes the probability of failure in the i-th dike subsection. By applying Eq. (14), the probability of failure of a dike section can be easily computed.

4 DIRECTIONAL SIMULATION

The aim of this section is to calculate the probabilities of dike failure due to uplifting and piping both for one subsection separately and for two adjacent subsections combined. In calculating these failure probabilities using standard Monte Carlo simulation, the problem arises that there are not enough samples in the failure region to obtain reliable results. To speed up Monte Carlo simulation, we use directional sampling.

Roughly speaking, directional simulation means the following. Rather than sampling straight from the probability distributions of the sea water level S, the river discharge Q, and the critical head H_{up}, we sample the directional angle and the directional radius in the (s, q, h_{up})-plain. This pays off when we have the fortune of being able to calculate the conditional probability that the length of the radius belongs to the failure region in explicit form. For example, when n random quantities have a multivariate normal distribution it is well-known that the conditional distribution of the squared radius, when the value of the directional vector is given, is a chi-square distribution with n degrees of freedom (see Ditlevsen & Madsen, 1996, Chapter 9).

The number of random quantities employed in the directional sampling program have been reduced to three on the basis of graphical steering. Cooke & Van Noortwijk (1999) visualised the effects of the residual random quantities using scatter plots. Using these plots they argued that the most important random quantities are the river discharge, the sea water level, and the critical head.

In order to calculate the probability of failure due to uplifting and piping, we now apply directional simulation to three standard exponentially distributed random quantities that are statistically independent. The reason for this is that we can easily transform the river discharge Q_{2days}, the sea water level S_{2days}, and the critical head in the event of uplifting and piping H_{up} to three standard exponentially distributed random quantities. This can be achieved by applying the transformation

$$\begin{aligned} \bar{F}_{Q_{2days}}(q) &= \exp\{-x\}, \\ \bar{F}_{S_{2days}}(s) &= \exp\{-y\}, \qquad (15) \\ \bar{F}_{H_{up}}(h) &= \exp\{-w\}, \end{aligned}$$

or, equivalently,

$$\begin{aligned} x &= -\ln(\bar{F}_{Q_{2days}}(q)) = r\sin(\theta)\sin(\psi), \\ y &= -\ln(\bar{F}_{S_{2days}}(s)) = r\cos(\theta)\sin(\psi), \qquad (16) \\ w &= -\ln(\bar{F}_{H_{up}}(h)) = r\cos(\psi), \end{aligned}$$

where r is the directional radius, and θ and ψ are the directional angles. Since the random quantities Q_{2days}, S_{2days}, and H_{up} are independent, the standard exponentially distributed random quantities X, Y, and W are also independent. Hence, since the Jacobian of transformation (16) equals $r^2\sin(\psi)$, the joint probability density function of the directional coordinates (R,Θ,Ψ) can be written as

$$f_{R,\Theta,\Psi}(r,\theta,\psi) = r^2 \sin(\psi)\exp\{-g(\theta,\psi)r\}\times \times I_{[0,\infty)}(r) I_{[0,\pi/2]}(\theta) I_{[0,\pi/2]}(\psi), \tag{17}$$

with

$$g(\theta,\psi) = \sin(\theta)\sin(\psi) + \cos(\theta)\sin(\psi) + \cos(\psi), \tag{18}$$

where $I_A(x) = 1$ if $x \in A$ and $I_A(x) = 0$ if $x \notin A$ for every set A. Accordingly, the joint probability density function of Θ and Ψ becomes

$$f_{\Theta,\Psi}(\theta,\psi) = \frac{2\sin(\psi)}{[g(\theta,\psi)]^3} I_{[0,\pi/2]}(\theta) I_{[0,\pi/2]}(\psi). \tag{19}$$

From the probability density function of the random vector (R,Θ,Ψ), and the probability density function of the random vector (Θ,Ψ), the conditional probability density function of R for fixed values of Θ and Ψ writes as

$$f_{R|\Theta,\Psi}(r \mid \theta,\psi) = = \frac{[g(\theta,\psi)]^3}{2} r^2 \exp\{-g(\theta,\psi)r\} I_{[0,\infty)}(r) \tag{20}$$

with cumulative distribution function

$$F_{R|\Theta,\Psi}(r \mid \theta,\psi) = = 1 - \left(1 + g(\theta,\psi)r + \frac{[g(\theta,\psi)r]^2}{2}\right)\exp\{-g(\theta,\psi)r\}. \tag{21}$$

The conditional distribution of R when $(\Theta,\Psi) = (\theta,\psi)$ is given, can be recognised as a gamma distribution with shape parameter 3 and scale parameter $g(\theta,\psi)$. Given the value of (Θ,Ψ), the probability of failure can now be written as

$$\Pr\{R > r^*(\theta,\psi) \mid \Theta = \theta, \Psi = \psi\} = = 1 - F_{R|\Theta,\Psi}(r^*(\theta,\psi) \mid \theta,\psi), \tag{22}$$

where $r^*(\theta,\psi)$ is the zero of the limit-state function $z(q,s,h)$ described in Section 2:

$$z(\bar{F}^{-1}_{Q_{2days}}(\exp\{-r^*(\theta,\psi)\sin(\theta)\sin(\psi)\}), \bar{F}^{-1}_{S_{2days}}(\exp\{-r^*(\theta,\psi)\cos(\theta)\sin(\psi)\}), \bar{F}^{-1}_{H_{up}}(\exp\{-r^*(\theta,\psi)\cos(\psi)\})) = 0. \tag{23}$$

Unfortunately, the zero $r^*(\theta,\psi)$ cannot be obtained in explicit form and has to be determined numerically.

In order to sample values of (Θ,Ψ), both the marginal probability density function of Θ and the conditional probability density function of Ψ, when the value $\Theta = \theta$ is given, remain to be determined. After some algebra, the marginal probability density function of Θ can be written as

$$\begin{aligned} f_\Theta(\theta) &= \int_{\psi=0}^{\pi/2} \frac{2\sin(\psi)}{[g(\theta,\psi)]^3} I_{[0,\pi/2]}(\theta)\, d\psi = \\ &= \int_{\tan(\psi)=0}^{\infty} \frac{2\tan(\psi)}{[v(\theta)\tan(\psi)+1]^3} I_{[0,\pi/2]}(\theta)\, d\tan(\psi) = \\ &= -\frac{I_{[0,\pi/2]}(\theta)}{[v(\theta)]^2} \cdot \left.\frac{2v(\theta)\tan(\psi)+1}{[v(\theta)\tan(\psi)+1]^2}\right|_{\tan(\psi)=0}^{\infty} = \\ &= \frac{1}{[v(\theta)]^2} I_{[0,\pi/2]}(\theta), \end{aligned} \tag{24}$$

where

$$v(\theta) = \sin(\theta) + \cos(\theta) \tag{25}$$

with cumulative distribution function

$$F_\Theta(\theta) = 1 - \frac{1}{1+\tan(\theta)}. \tag{26}$$

Accordingy, the conditional probability density function of Ψ when the value $\Theta = \theta$ is given follows from Eqs. (19) and (24):

$$f_{\Psi|\Theta}(\psi \mid \theta) = \frac{2\sin(\psi)[v(\theta)]^2}{[g(\theta,\psi)]^3} I_{[0,\pi/2]}(\psi) \tag{27}$$

with cumulative distribution function

$$F_{\Psi|\Theta}(\psi \mid \theta) = \frac{1}{f_\Theta(\theta)} \int_{\tilde{\psi}=0}^{\psi} f_{\Theta,\Psi}(\theta,\tilde{\psi})\, d\tilde{\psi} = = 1 - \frac{2v(\theta)\tan(\psi)+1}{[v(\theta)\tan(\psi)+1]^2}. \tag{28}$$

Let P_1 and P_2 be independent and standard uniformly distributed random quantities, then values of the random vector (Θ,Ψ) can be generated as follows:

$$\theta = \arctan\left(\frac{p_1}{1-p_1}\right), \quad \psi = \arctan\left(\frac{1}{[\sin(\theta)+\cos(\theta)]} \cdot \frac{\sqrt{p_2}}{1-\sqrt{p_2}}\right). \tag{29}$$

Note that for two-dimensional directional sampling, the probability density function of the directional angle Θ is also Eq. (24). The conditional distribution of the directional radius R, when the directional angle θ is given, is a gamma distribution with shape parameter 2 and scale parameter $v(\theta)$.

5 RESULTS AND CONCLUSIONS

In this section, the results are presented for dike section 13.1 of the Dutch dike ring the Hoeksche Waard. The probability of dike failure due to uplifting and piping per two days can be calculated using

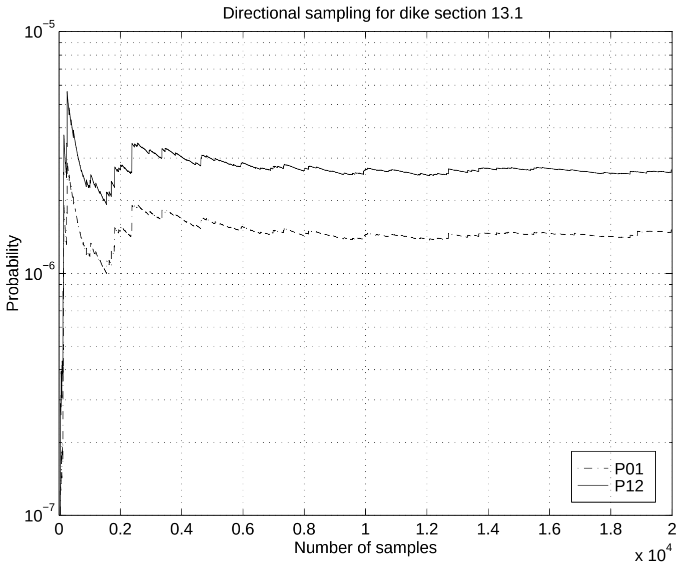

Figure 2. The probability of failure of one subsection ('P01') and two subsections ('P12') of dike section 13.1 determined on the basis of 20,000 samples using three-dimensional directional sampling.

directional sampling on the basis of the sea water level, the river discharge, and the critical head in the event of uplifting and piping.

Furthermore, the spatial variation and correlation of the critical head in the event of uplifting and piping can be modelled using a Markovian dependency structure. This means that the random quantities, representing the inherent uncertainties in the critical head, of one dike subsection only depend on the values of the corresponding random quantities in the two adjacent dike subsections.

The results of the three-dimensional directional simulation (on the basis of 20,000 samples) and the subsequent dike ring reliability calculations are as follows:

- the expected probability of failure due to uplifting and piping of one subsection of length $x^* = 50$ metres is $p(x^*) = 1.65\ 10^{-6}$;
- the expected probability of failure due to uplifting and piping of two adjacent subsections is $p(2x^*) = 2.76\ 10^{-6}$;
- since the length of dike section 13.1 is $l = 100$ metres, the expected probability of failure due to uplifting and piping of the dike section 13.1 $p(l) = 2.76\ 10^{-6}$ (note that if dike section 13.1 would be 1,000 metres of length, Eq. (14) would lead to a failure probability of $2.27\ 10^{-5}$).

The iteration process which resulted in this failure probability is shown in Figure 2. It clearly illustrates how fast the three-dimensional directional simulation converges! The advantage of three-dimensional directional sampling is that large sample sizes are not required (sample sizes of about 20,000 samples already supply satisfactory results). The directional simulation results above have been compared to results obtained using First Order Reliability Method. According to Vrouwenvelder (1998), FORM results in a probability of failure due to uplifting and piping of $2.1\ 10^{-6}$. Hence, both results are quite close to each other.

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