Bayesian estimation of return periods of CSO volumes for decision-making in sewer system management

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Keywords

Sewer system, probabilistic modelling, Bayesian analysis

Abstract

Decisions on the rehabilitation of a sewer system are usually based on a single computation of CSO volumes using a time series of rainfall as system loads. A shortcoming of this method is that uncertainties in knowledge of sewer system dimensions are not taken into account. Besides, statistical uncertainties are left aside. This paper presents the effect of variations in sewer system dimensions on return periods of calculated CSO volumes. As an example the sewer system of 'De Hoven' (the Netherlands) is used. CSO volumes per storm event are computed using Monte Carlo simulations with a reservoir model of the sewer system. In each Monte Carlo run random values for the sewer system dimensions are drawn and substituted in the model. With regard to the computed CSO volumes probability distributions are estimated taking into account the statistical uncertainties involved. For this purpose so-called Bayes factors are used to determine weights that describe how well a probability distribution fits the computed data, i.e. the better the fit, the higher the weighing. With the fitted probability distributions the 95% uncertainty intervals of calculated CSO volumes and their corresponding return periods are computed. The results show that uncertainties in knowledge of sewer system dimensions cause a considerable variability in return periods of calculated CSO volumes.

Introduction

Decisions on sewer system rehabilitations are taken under substantial uncertainties, including uncertainties in knowledge of sewer system dimensions. As a result the effectiveness of investments in sewer system rehabilitation may be questioned. For example, in the Netherlands a number of examples are known in which the rehabilitations did not have the desired effect.

In current practice, assessment of sewer system performance is based on return periods of CSO (Combined Sewer Overflow) volumes and flooding events. However, sufficiently long time series of measurements of for example CSO volumes are usually not available for a sewer system. Generating a series of CSO volumes with a model of the sewer system solves this problem of data scarcity. The model requires data on sewer system dimensions (e.g. storage volume, pumping capacity, contributing areas, etc) and hydraulic loads (precipitation and dry weather flow (dwf)). Since only one model run is performed, uncertainties in knowledge of sewer system dimensions are not considered. As a result, the effect of uncertainties in system dimensions on the return period of calculated CSO volumes remains unknown.

For quantification of the influence of variations in system dimensions on the return period of calculated CSO volumes the probability distribution of calculated CSO volumes needs to be estimated. A practical difficulty in fitting probability distributions to CSO data is that often only a limited amount of data is available. If only sparse data is available more than one distribution seems to fit the observed data and only a few can be rejected on the basis of probability plots or goodness-of-fit tests (e.g. Chi-square or Kolmogrov-Smirnov). Uncertainty about the distribution type and the parameters of the distribution comprise the statistical uncertainty. As an alternative, a Bayesian approach can be used to determine how well a probability distribution fits observed data. Bayesian estimation takes into account statistical uncertainties involved. This kind of distribution type selection has been applied to civil engineering problems by e.g. Van Gelder *et al.* (1999), Chbab *et al.* (2000), Van Gelder (2000) and Van Noortwijk *et al.* (2001). For applications of probabilistic modelling in the field of urban drainage, we refer to Novotny and Witte (1997), Reichert (1997), Willems and Berlamont (1999) and Willems (2001).

This paper discusses the variability in return periods of calculated CSO volumes due to uncertainties in knowledge of sewer system dimensions. For this purpose a Bayesian method for estimating the probability distribution of calculated CSO volumes is presented. At first, statistical uncertainties are treated. Subsequently, Bayesian estimation in general and a Bayesian approach for selecting probability models using so-called Bayes weights are considered. The effect of uncertainties in sewer system dimension on return periods of calculated CSO volumes is studied in a case study. Return periods are computed using Bayesian estimation. The paper ends with conclusions.

Statistical uncertainties

Types of uncertainty. According to Van Gelder (2000) uncertainties in decision and risk analysis can primarily be divided in two categories (Figure 1):

- Inherent uncertainty: uncertainties that originate from variability in known (or observable) populations and therefore represent randomness in samples (e.g. measured rainfall volumes).
- Epistemic uncertainty: uncertainties that originate from lack of knowledge of fundamental phenomena (e.g. rainfall-runoff process).

Inherent uncertainties represent randomness or variability in nature (Figure 1). For example, even in the event of sufficient data, one cannot predict the maximum rain intensity that will occur next year. The two main types of inherent uncertainty are inherent uncertainty in time (e.g. variations of rainfall intensities in time) and inherent uncertainty in space (e.g. fluctuations in local terrain slope).

Epistemic uncertainties represent the lack of knowledge about a physical system, e.g. limited knowledge about in-sewer processes (Ashley *et al.*, 1998) (Figure 1). The two main types of epistemic uncertainty are model uncertainty (due to lack of understanding of the physics) and statistical uncertainty (due to lack of sufficient data). Model uncertainty is subdivided into model parameter and model structure uncertainties, statistical uncertainty into statistical parameter and distribution uncertainties.

In general, epistemic uncertainties can be reduced as knowledge increases and more data becomes available.

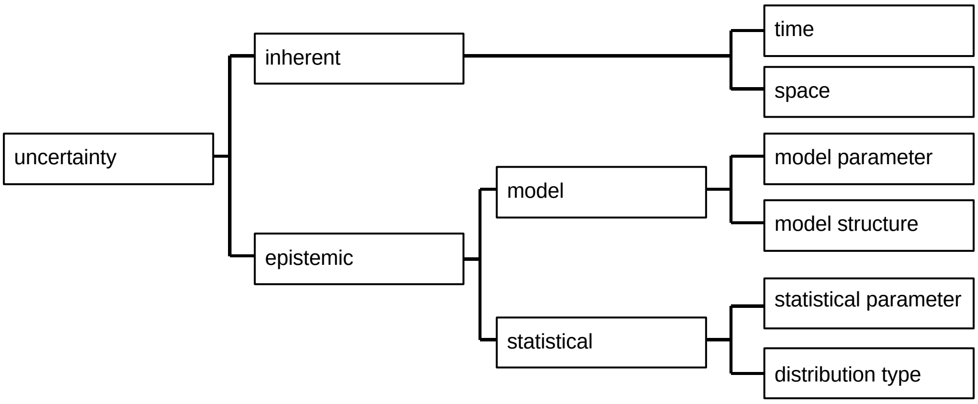

Figure 1. Types of uncertainty. Uncertainties can primarily be divided in inherent and epistemic uncertainty (Van Gelder, 2000). The latter consists of model and statistical uncertainty.

Statistical parameter and distribution type uncertainty. Statistical uncertainty may contribute considerably to the overall uncertainty. The uncertainty caused by the fact that the parameters of a distribution (e.g. normal, exponential and lognormal) are determined with a limited number of data is called statistical parameter uncertainty. Statistical parameter uncertainty can be calculated by means of bootstrapping or Bayesian methods (Van Gelder, 2000).

In addition to statistical parameter uncertainty, statistical distribution uncertainty is of importance. This type of uncertainty represents the uncertainty of the distribution type of a variable. For example, beforehand it is not clear whether rainfall intensities are exponentially or lognormally distributed or have another distribution. Besides, if only sparse data is available more than one distribution seems to fit the observations and only a few can be rejected on the basis of goodness-of-fit tests (Chi-square and Kolmogorov-Smirnov). Bayesian selection methods can be used to avoid this problem (Van Gelder, 2000 and Van Noortwijk *et al.*, 2001).

Bayesian estimation

Bayesian statistics is the only statistical theory that combines modelling inherent uncertainty and statistical uncertainty. The theory of Bayesian statistics is described in detail in a number of textbooks such as Benjamin and Cornell (1970). The theorem of Bayes (1763) provides a solution to the problem of how to learn from new data. Bayes' theorem (i.e. the conditional probability theorem) is written as,

$$p(q|\mathbf{x}) = \frac{\ell(\mathbf{x}|q)p(q)}{\int_{\Theta} \ell(\mathbf{x}|q)p(q)\,dq} = \frac{\ell(\mathbf{x}|q)p(q)}{p(\mathbf{x})}, \tag{1}$$

where ð(*è*|**x**) is the posterior density of è = ($è_1$,…,$è_d$) after observing data **x** = (x_1,…,x_n), ℓ(*è*|**x**) is the likelihood function of observations **x** = (x_1,…,x_n) when the parameter è = ($è_1$,…,$è_d$) is known, ð(*è*) is the prior density of è = ($è_1$,…,$è_d$) before observing data **x** = (x_1,…,x_n) and ð(**x**) is the marginal density of the observations **x** = (x_1,…,x_n).

If X has a probability density function ℓ(*x*|*è*), then the likelihood function of the independent observations **x** = (x_1,…, x_n) is given by,

$$\ell(\mathbf{x}|q) = \ell(x_1, x_2, \ldots, x_n|q) = \prod_{i=1}^{n} \ell(x_i|q). \tag{2}$$

The likelihood function of the observations $\ell(\mathbf{x}|\theta)$ represents the inherent uncertainty of a random variable X when θ is given. Statistical uncertainty in θ is represented in the prior density $\pi(\theta)$ and the posterior density $\pi(\theta|\mathbf{x})$. Both statistical uncertainties are parameter uncertainty.

Using Bayes' theorem a prior distribution can be updated as soon as new observations are available. The more new observations are used, the smaller the parameter uncertainty in θ. In other words, Bayes' theorem updates subjective beliefs on the occurrence of an event based on new data. With the assessment of sewer system performance one is interested in the probability of exceeding a certain CSO volume x_0. The posterior predictive probability of exceeding x_0 is calculated from the survival function of X (i.e. probability of exceeding x given parameter vector θ), which is denoted as,

$$\bar{F}(x|\theta) = 1 - F(x|\theta) = 1 - \Pr\{X \le x|\theta\} = \int_{x}^{\infty} f(t|\theta)\,dt. \quad (3)$$

This gives a posterior predictive probability of exceedance equal to

$$\Pr\{X > x_0|\mathbf{x}\} = \int_{\theta} \Pr\{X > x_0|\theta\}\pi(\theta|\mathbf{x})\,d\theta = \int_{\theta} \bar{F}(x_0|\theta)\pi(\theta|\mathbf{x})\,d\theta, \quad (4)$$

where $\Pr\{X > x_0|\mathbf{x}\}$ is the predictive probability of exceeding x_0 when the observations $\mathbf{x} = (x_1,\dots,x_n)$ are given.

Bayesian statistics can not only represent statistical parameter uncertainty, but also take into account distribution type uncertainty using Bayes factors or Bayes weights.

Bayes factors and Bayes weights

In decision-making for sewer system management the question arises which probability distribution should be chosen to model the performance parameter, i.e. CSO volumes. Instead of choosing one probability distribution type one could also consider various possible distributions and attach weights to the distributions according to how good the fits of these distributions are. Weight factors for probability distributions can be determined with different methods (see Van Gelder, 2000).

Hypothesis testing. The traditional approach would be to formulate two probability models (or hypotheses) H_1 and H_2. A test statistic (e.g. χ^2 test) is used to judge whether hypothesis H_1 should be rejected or not. Probability model H_1 is rejected if the test statistic is smaller than a certain value, which was determined beforehand. The traditional approach of model testing has quite a few disadvantages. It can only be applied if two models are nested. Besides, it can only offer evidence against a hypothesis H_1 or the alternative H_2. Acceptance of hypothesis H_1 on the basis of the traditional approach is not possible.

Bayesian hypothesis testing. Van Gelder (2000) and Van Noortwijk *et al.* (2001) use a Bayesian method to choose one probability model from a set of possible models. The disadvantages of the traditional approach do not exist in this Bayesian hypothesis testing. The number of candidate models that can be considered simultaneously is not limited. Moreover, models do not have to be nested (one within another). The Bayesian approach to hypothesis testing originates from the work of the physicist Sir Jeffreys (Jeffreys, 1961). It is a methodology for quantifying the evidence in favour of a scientific theory using Bayes factors. The approach quantifies statistical uncertainty. Kass and Raftery (1995) give a recent overview of Bayes factors.

Consider a data set $\mathbf{x} = (x_1,\dots,x_n)$ and two candidate probability models H_1 and H_2. The two hypotheses H_1 and H_2 represent two marginal probability densities $\pi(\mathbf{x}|H_1)$ and $\pi(\mathbf{x}|H_2)$.

Given the prior probabilities $p(H_1)$ and $p(H_2)$ = 1-$p(H_1)$ the data produce posterior probabilities $p(H_1|\mathbf{x})$ and $p(H_2|\mathbf{x})$ = 1-$p(H_1|\mathbf{x})$. When the two hypotheses are considered equally probable beforehand, $p(H_1) = p(H_2) = 0.5$ are chosen.

The posterior probabilities are obtained using Bayes' theorem,

$$p(H_k|\mathbf{x}) = \frac{p(\mathbf{x}|H_k)p(H_k)}{p(\mathbf{x}|H_1)p(H_1) + p(\mathbf{x}|H_2)p(H_2)} \qquad k = 1,2. \tag{5}$$

These probabilities are called Bayes weights, i.e. the posterior probability of model H_k being correct given the data $\mathbf{x} = (x_1,\ldots,x_n)$. The marginal density of the data ð($\mathbf{x}|H_k$) under model H_k is obtained by integrating with respect to the parametric vector è$_k$,

$$p(\mathbf{x}|H_k) = \int \ell(\mathbf{x}|q_k, H_k)p(q_k|H_k)dq_k, \tag{6}$$

where ð(è$_k|H_k$) is the prior density of H_k and ℓ($\mathbf{x}|$è$_k,H_k$) is the likelihood function of the data $\mathbf{x}$ given è$_k$. The results can be summarised in the so-called Bayes factor,

$$B_{12} = \frac{p(H_1|\mathbf{x})/p(H_2|\mathbf{x})}{p(H_1)/p(H_2)}, \tag{7}$$

which can be reduced using Bayes' theorem to,

$$\frac{p(\mathbf{x}\ H)}{p(\mathbf{x}\ H)}. \tag{8}$$

m candidate models $_k$ (= 1,…,m

$$p(H_k|\mathbf{x}) = \frac{p(\mathbf{x}|H_k)p(H_k)}{\sum_{j=1}^{m} p(\mathbf{x}|H_j)p(H_j)} \qquad k = 1,\ldots,m, \tag{9}$$

which results in Bayes factors defined as,

$$B_{jk} = \frac{p(\mathbf{x}|H_j)}{p(\mathbf{x}|H_k)} \qquad j,k = 1,\ldots,m. \tag{10}$$

Non-informative priors. For the purpose of predicting return periods of CSO volumes, one would like the observed or computed volumes to 'speak for themselves'. This means that a prior distribution should describe a certain 'lack of knowledge'. For this purpose so-called non-informative priors have been developed. Non-informative priors represent the type of information to be used for making the observations dominant when a particular likelihood model is given. According to Van Noortwijk *et al.* (2001) the Jeffreys prior is considered to be most appropriate for purposes of 'fully objective' formal model comparison.

A disadvantage of non-informative priors is that these priors are improper, or in other words they do not integrate to one. Because of this the Bayes factors in (8) are undefined. The prior probability $p(H_k)$ is defined as,

$$p(H_k) = w(H_k)\int J(q_k|H_k)dq_k, \tag{11}$$

where $w(H_k)$ is the prior weight of probability model H_k and J(è$_k|H_k$) is the integral over the non-informative Jeffreys prior, which is often infinite. The problem is resolved by defining the Bayes factors differently,

$$\frac{p(H_j|\mathbf{x})}{p(H_k|\mathbf{x})} = \frac{p(H_j|\mathbf{x})}{p(H_k|\mathbf{x})} \times \frac{w(H_j)}{w(H_k)} \qquad j,k = 1,\ldots,m. \tag{12}$$

Using (9) and (12) the posterior probability of model H_k being correct (i.e. the Bayes weight) can be rewritten as,

$$p\left(H_k \mid \mathbf{x}\right) = \frac{p\left(\mathbf{x} \mid H_k\right) w\left(H_k\right)}{\sum_{j=1}^{m} p\left(\mathbf{x} \mid H_j\right) w\left(H_j\right)} \qquad k = 1, \ldots, m. \tag{13}$$

It remains to choose the prior weights $w(H_k)$. For formal model comparison Van Noortwijk *et al.* (2001) propose to use equal prior weights, $w(H_k) = 1/m$, $k=1,\ldots,m$.

When using the improper Jeffreys prior the marginal density of the data given in (6) is difficult to compute. A solution is to approximate the logarithm of the marginal density with the Laplace expansion (Van Noortwijk *et al.*, 2001),

$$\log\left(p\left(\mathbf{x} \mid H\right)\right) \approx \frac{d}{2}\log\left(2p\right) - \frac{d}{2}\log\left(n\right) + \log \ell\left(\mathbf{x} \mid \hat{q}, H\right) \tag{14}$$

for $n \to \infty$, where $\hat{q}$ is the maximum likelihood estimator under model H, d is the number of parameters in model H and n is the number of observations. The second and third term on the right-hand side of Eq.(14) form the so-called Schwartz Criterion for model selection. Despite the fact that the relative error in the Bayes factor using the Laplace expansion has an accuracy of $O(1)$, the approximation appears to work rather well in practice.

Case study: Return period analysis of sewer system 'De Hoven'

The influence of variations in system dimensions, such as storage capacity and contributing areas, on the return period of calculated CSO volumes is studied by modelling the sewer system of 'De Hoven' (Clemens, 2001). The catchment 'De Hoven' (12.69ha) is situated in the Netherlands on the banks of the river IJssel in the city of Deventer. The sewer system ($865 m^3$) is of the combined type and comprises one pumping station ($119 m^3/h$) transporting the sewage to a treatment plant and three CSO structures. The sewer system is modelled as a reservoir with an external weir and a pump. The rainfall runoff is modelled with the so-called NWRW 4.3 model (Figure 2), the standard rainfall runoff model in the Netherlands. In this model evaporation, infiltration, storage on street surfaces and overland flow are modelled as described in e.g. Clemens (2001).

Table 1. Variations in system parameters (from: Clemens, 2001).

System parameter	ì	ó	CV (%)
S (m^3)	865.0	43.25	5.0
pc (m^3/h)	119.0	5.95	5.0
A (ha)	12.69	0.64	5.0
CC ($m^{0,5}/s$)	1.40	0.35	25.0

The influence of variability in four sewer system dimensions is studied: storage volume (S), pumping capacity (pc), contributing area (A) and overflow coefficient (CC). These dimensions are assumed to be normally distributed with known μ and σ and independent. Averages and standard deviations are based on expert judgement (Clemens, 2001). As input of the computations a 10-year rainfall series (1955-1964) of KNMI (De Bilt, the Netherlands) is used.

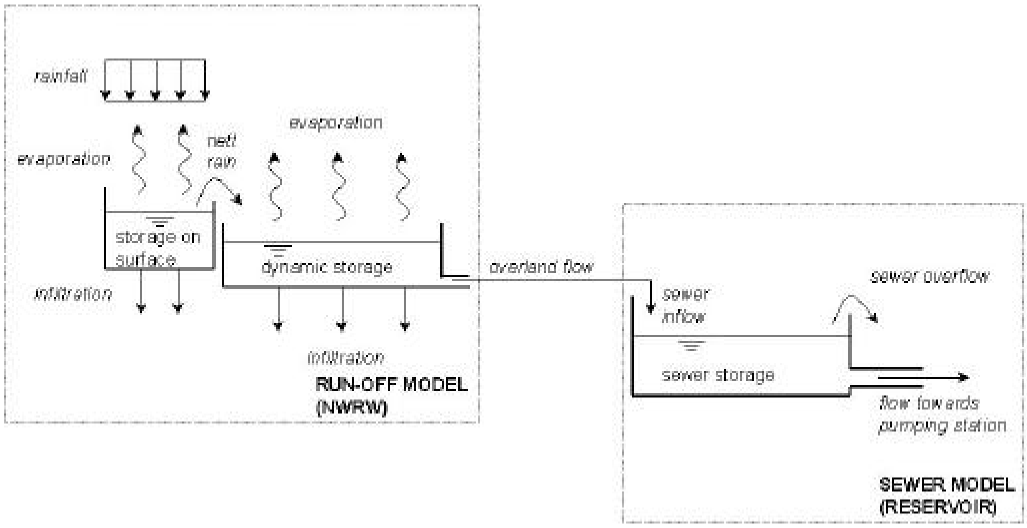

Figure 2. Model of sewer system. The model comprises a rainfall runoff model and a reservoir model with an external weir and a pump.

A Monte Carlo simulation with 1000 runs is performed. In each run a random value of the model parameters (S, pc, A, CC) is drawn from the probability distribution functions. The random samples are substituted in the reservoir model. The four parameter values are drawn independently, since their covariances are equal to 0 in the reservoir model. Otherwise, a simulation scheme based on a multi-variate normal distribution (Cholesky decomposition) can be considered. The computed CSO volumes are summed over the storm events. A storm event is defined as an event that starts when rainfall occurs resulting in a water level rise in the sewer system above dwf level. The event lasts until the water level drops below dwf level again. This results in statistically independent storm events and CSO volumes per storm event.

The uncertainties in the computed CSO volumes comprise not only model parameter uncertainty due to the variation in system parameters, but also model structure uncertainty due to the strongly simplified reservoir model and inherent uncertainty in time due to the temporal variation in rainfall. Model structure uncertainty decreases when a more detailed model is applied given sufficient data are available for model calibration.

Table 2. Prior and posterior Bayes weights for calculated CSO volumes per storm event for sewer system 'De Hoven'.

Bayes weight	Exponential	Rayleigh	Normal	Lognormal	Gamma	Weibull	Gumbel
Prior	0.1429	0.1429	0.1429	0.1429	0.1429	0.1429	0.1429
Posterior	0.0766	0.0000	0.0000	0.0087	0.4089	0.5059	0.0000

Using Bayes weights the distribution function with the best fit to the CSO data is chosen. Exponential, Rayleigh, normal, lognormal, gamma, Weibull and Gumbel distributions are considered. The Bayes weights are computed for 10 randomly selected runs from the Monte Carlo simulation with 1000 runs. In the computation of the weights, Jeffreys priors are used as prior distributions and the Laplace expansion is used for approximation of marginal densities. In Table 2 the averages of the prior and posterior Bayes weights of these 10 runs are given. The Weibull distribution appears to fit best with a Bayes weight of 51%. Therefore, the Weibull distribution is chosen to describe the calculated CSO volumes per storm event.

A random variable X has a Weibull distribution with shape parameter a>0 and scale parameter b>0 if probability density is given by,

$$f(x) = \frac{a}{b}\left(\frac{x}{b}\right)^{a-1} \exp\left[-\left(\frac{x}{b}\right)^{a}\right]. \tag{15}$$

The corresponding survival function of the Weibull distribution (i.e. the probability of exceedance) is defined as,

$$\bar{F}(x) = \exp\left[-\left(\frac{x}{b}\right)^{a}\right]. \tag{16}$$

Given the data $\mathbf{x} = (x_1,\ldots,x_n)$ the shape parameter a and the scale parameter b of a Weibull distribution can be estimated with the Maximum Likelihood (ML) method. As an alternative, the parameters could be estimated using Bayes' theorem. As the number of observations n approaches infinity, a ML estimate is similar to a Bayesian estimate with non-informative priors. However, the ML estimator produces a point estimate of parameters a and b, whereas a Bayesian analysis gives the probability densities of these parameters. The Bayes estimates of a and b are the posterior means of the posterior distributions of a and b, respectively.
For the sake of convenience, a ML estimate is used. The fully Bayesian approach will be applied in a forthcoming paper. The loglikelihood of the Weibull distribution is,

$$\log \ell(\mathbf{x}|a,b) = n\{\log(a) - a\log(b)\} + (a-1)\sum_{i=1}^{n}\log\{x_i\} - \sum_{i=1}^{n}\left(\frac{x_i}{b}\right)^{a}. \tag{17}$$

With the Maximum Likelihood method those values of a and b are chosen for which the likelihood function (2) is maximised. Consequently, the maximum likelihood estimator of parameter vector (a,b) is defined as,

$$\left(\hat{a},\hat{b}\right) = \max_{(a,b)} \ell(\mathbf{x}|a,b) = \max_{(a,b)} \log\left[\ell\{\mathbf{x}|(a,b)\}\right] \tag{18}$$

Computation of return periods of calculated CSO volumes requires not only estimating the probability of threshold exceedances (i.e. CSO volumes per event), but also specifying the stochastic process of the occurrence times of these exceedances. The threshold exceedances are assumed to be mutually independent and to have a Weibull distribution. The occurrence process of exceedances of the threshold can be regarded as a Poisson process (see e.g. Buishand, 1989). As a result, the return period of a CSO volume depends on the exceedance probability of this volume and the average return period of exceedances (irrespective of the volume),

$$\frac{1}{T_{V_{CSO}>v_0}} = \frac{1}{T_{CSO}} * \Pr\{V_{CSO} > v_0|\mathbf{x}\} = \frac{1}{T_{CSO}}\left[1-\Pr\{V_{CSO} \le v_0|\mathbf{x}\}\right] = \frac{1}{T_{CSO}}\left[1-F\{v_{CSO}\}\right], \tag{19}$$

where $T_{Vcso>v_0}$ is the return period of calculated CSO volumes (V_{CSO}) larger than v_0, T_{CSO} is the average return period of CSOs irrespective of their volume, $\Pr\{V_{CSO}>v_0|\mathbf{x}\}$ is the probability of exceeding volume v_0 given data set $\mathbf{x} = (x_1,\ldots,x_n)$ of computed CSO volumes and $F(v_{CSO})$ is the cumulative distribution function of CSO volumes. The average return period of calculated CSOs irrespective of the volume is,

$$T_{CSO} = \frac{\#\,\text{years (over which CSO events are measured)}}{\#\,\text{CSO events}}, \tag{20}$$

Figure 3 shows calculated CSO volumes per storm event and their corresponding return periods. Both the average CSO volume and the 95% uncertainty interval are displayed. The figure is based on the performed Monte Carlo simulation with 1000 runs. It demonstrates that calculated CSO volumes with a certain return period show considerable variation. For example, in the event of a return period of 0.5y the average calculated CSO volume is 6.1mm and the 95% uncertainty interval of this volume is 2.7mm. However, with a return period of 5y the average increases to 22.0mm and the 95% uncertainty interval to 6.2mm.

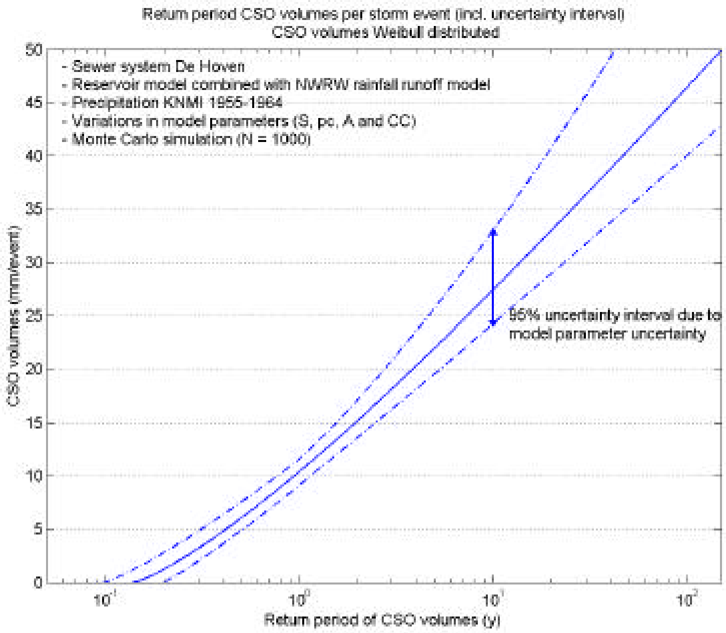

Figure 3. Return period of calculated CSO volumes per storm event (including 95% uncertainty interval). The CSO volumes per storm event are Weibull distributed.

Conclusions

Return periods of calculated CSO volumes are an important criterion in decision-making for sewer system rehabilitation. In this paper, the variability in return periods of CSO volumes of the sewer system 'De Hoven' is determined using Bayes weights. Determination of the distribution type of a data set with Bayes weights takes into account statistical uncertainties, which stem from lack of data. Using Bayes weights enables discrimination between different probability models and quantification of the fit between the distributions and the data. With respect to variability in the return periods of CSO volumes in 'De Hoven' the following conclusions are drawn:

- Bayes weights have been successfully applied to estimate return periods of calculated CSO volumes taking into account statistical uncertainties involved.
- Calculated CSO volumes per storm event are Weibull distributed.
- For a certain return period calculated CSO volumes show considerable variation due to uncertainties in knowledge of sewer system dimensions. The variation increases with increasing return periods.

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