Sampling inspection for the evaluation of time-dependent reliability of deteriorating structures

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ABSTRACT: The paper presents a sampling-inspection strategy for the evaluation of time-dependent reliability of deteriorating structures, where the deterioration is assumed to initiate at random times and at random locations. After initiation, defects are weakening the structure's resistance. The system becomes unacceptable when at least one defect reaches a critical depth. The defects are assumed to initiate at random times modeled as event times of a Non-Homogeneous Poisson Process (NHPP) and to develop according to a non-decreasing time-dependent stationary gamma process. The intensity rate of the NHPP is assumed to be a combination of a known shape function in time and an unknown proportionality constant. When sampling inspection (i.e., inspection of a selected subregion of the structure) results in the number of defect initiations, Bayes' theorem can be used to update prior beliefs about the proportionality constant of the NHPP intensity rate to the posterior distribution. On the basis of a time- and space-dependent Poisson process for the defect initiation, an adaptive Bayesian model for sampling inspection is developed to determine the predictive probability distribution of the time to failure. A potential application is, for instance, the inspection of a large vessel or pipeline suffering pitting/localized corrosion in the oil industry.

1 INTRODUCTION

In the oil industry, corrosion is the main threat for inspection engineers responsible for the overall safety of the industrial plants. Inspection reveals damage caused by corrosion and helps in judging the system safety and its capability of future functioning. In order to justify the safety and functioning, one of the options is to inspect the entire system. However, such a complete inspection is not always feasible or necessary and may be too costly. Instead, the so-called *sampling inspection* is used. Sampling inspection is a partial inspection of the system, where only selected sections are inspected. The overall system safety is then judged based on the results from the inspected parts.

We present a Bayesian method for inference on the number of defects in the entire system based on partial inspection. We assume a deterioration process of local defects weakening the structural resistance (e.g. pitting corrosion) and an inspection counting all the defects present in the inspected parts of the system.

This paper is organized as follows. Section 2 presents the mathematical formulation of the deterioration process, which is a combination of the following two stochastic processes: the defect initiation process modeled as event times from a Non-Homogeneous Poisson Process (NHPP) and the defect growth process modeled by a stationary gamma process. In Section 3 the sampling inspection is described, where the entire system is subdivided into the inspected sections. We use the so-called Bernoulli splitting mechanism to split the Poisson process of the initiation of all defects in the system to a number of Poisson processes that govern the initiation of defects in disjoint sections. Section 4 presents the evaluation of the failure probability for the entire system which is defined as the probability that in the operational time $(0, t]$ at least one defect occurs that is deeper than the critical depth or the corrosion allowance. An adaptive Bayesian model is developed to update the probability distribution of the proportionality constant of the NHPP intensity rate on the basis of partial observation of the number of defects in the system. Using the updated distribution, the Bayes estimate of the total number of defects can be obtained. Finally, Section 5 presents an example of the sampling inspection and Section 6 draws conclusions regarding the model and its applicability.

2 DETERIORATION PROCESS

In this paper, we focus on the deterioration process which initiates in space and time before weakening the system resistance. The goal is to combine the two stochastic processes of defect initiation and defect growth. This is done in the subsections below, where the Poisson process of defect initiation, the gamma process of wall penetration, and the localized corrosion process are defined and presented.

2.1 *The stochastic process of defect initiation*

It is unlikely that all defects appear at the same time. They rather initiate at random times and then grow depending on the environment and conditions of the component. For that reason, we model the appearance of the defects in time and the total number of defects that have initiated up to a certain time t. To achieve this, we assume that the number of defects in time follows a NHPP with certain intensity function.

The Poisson process is commonly used in applications. For instance, Nicolai et al. (2007) used the NHPP with power law intensity function to model the arrivals of localized corrosion on a steel structure with coating protection, and van Noortwijk & Klatter (1999) used a homogeneous Poisson process to model the initiation of scour erosion of the sea-bed protection of a storm-surge barrier.

In this paper we do not restrict ourselves to the power law intensity function. However, we assume that the defect initiations occur according to a NHPP with intensity function $\lambda m(t)$, where λ is the proportionality constant and $m(t)$ is an arbitrary intensity function. The expected number of defects up to time t is then:

$$E\left(N(t)\middle|\lambda\right)=\lambda\int_0^t m(s)ds=\lambda M(t)\,. \tag{1}$$

The interpretation of the expectation in Equation (1) is that given the value of the proportionality constant λ the intensity function is known and the number of defects follows a known Poisson distribution. However, depending on the value of λ different Poisson distributions of the number of defects are realized. Figure 1 shows two NHPP examples with proportionality constants λ = 1 and λ = 0.4 together with their realizations.

The reason for performing a statistical analysis solely on the NHPP proportionality constant is the usual lack of deterioration data. In this way, engineering knowledge about the shape of the expected number of defect initiations and the observed number of defect initiations can be combined. We assume that information about the shape of the intensity function $m(t)$ is known from physics and/or expert judgment. In this article, we apply Bayesian inference to estimate the proportionality constant λ leaving the function $m(t)$ as known.

In general, the arrival times from a NHPP are not independent. Suppose that the random vector $(S_1,\ldots,S_n)$ represents the first n arrival times from the NHPP with intensity function $m(t)$. It follows that the conditional joint probability density function of the vector $(S_1,\ldots,S_n)$ given $\{N(t)=n\}$ is:

$$f_{S_1,\ldots,S_n|N(t)}\left(s_1,\ldots,s_n\middle|n\right)=\frac{n!\prod_{i=1}^{n}m\left(s_i\right)}{\left[M(t)\right]^n}, \tag{2}$$

where $0<s_1<\ldots<s_n\le t$. It can be shown that conditional on $\{N(t)=n\}$, the vector $(S_1,\ldots,S_n)$ has the same distribution as the order statistic of n independent, identically distributed (*iid*) random variables U_i, where:

$$P\{U_i<s\}=\frac{M(s)}{M(t)}, \tag{3}$$

for $0\le s\le t$, which implies the density in Equation 2 (Kulkarin 1995, page 228, Theorem 5.12).

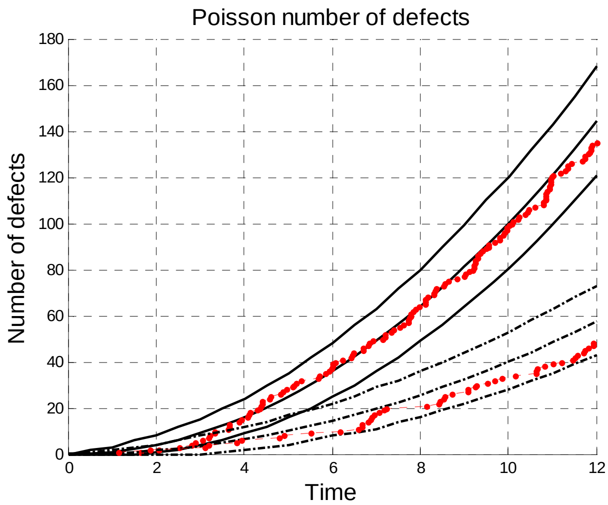

Figure 1: The expected value of the number of defects together with the 2.5% and 97.5% quantile of the NHPP with power law intensity $M(t)=t^2$ for λ = 1 (–) and λ = 0.4 (–·–).

2.2 *The stochastic process of defect growth*

Given that a defect has occurred, the process of wall penetration is activated. The wall thickness loss on a corroded spot is an increasing process in time and because of its monotonic and uncertain behavior a gamma stochastic process is proposed in the literature as a proper representation (Abdel-Hameed 1975, van Noortwijk 2007).

The gamma process is a stochastic process with independent gamma distributed increments making it always monotonic. More precisely: let $\alpha(t)$ be a non-decreasing, right continuous, real-valued function for $t \geq 0$, with $\alpha(0) = 0$. The gamma process with shape function $\alpha(t) > 0$ and scale parameter $\beta > 0$ is a continuous-time stochastic process $\{X(t)$: $t \geq 0\}$ with the following properties:

I. $X(0) = 0$ with probability one;

II. $X(t) - X(r) \sim ga(\bullet|\alpha(t) - \alpha(r), \beta)\ \forall t > r \geq 0$;

III. $X(t)$ has independent increaments;

where $ga(\cdot|\alpha, \beta)$ is the gamma probability density function. Recall that a random variable X has a gamma distribution with shape parameter $\alpha > 0$ and scale parameter $\beta > 0$ if its probability density function is given by:

$$ga(x|\alpha, \beta) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} x^{\alpha-1} \exp\{-\beta x\} \tag{3}$$

where $x > 0$.

In the literature on corrosion for steel structures, the assumption is often made that the corrosion rate (the rate of wall thickness loss) in the long run is constant in time (American Petroleum Institute 2000). Therefore, we restrict ourselves to the wall loss process represented by a stationary gamma process $\{X(t)$: $t \geq 0\}$ with linear shape function αt. In general, this restriction is not required and an arbitrary shape function $\alpha(t)$ can be used. It follows that the expectation and variance of the process $X(t)$ are linear in time and given by:

$$E(X(t)) = \frac{\alpha t}{\beta}, \quad Var(X(t)) = \frac{\alpha t}{\beta^2}. \tag{4}$$

The parameters α and β may be chosen based on the assessment of the expected annual wall thickness loss on a corrosive location and its variation.

In this paper, we do not discuss updating the corrosion growth process. It is assumed that the uncertainty in the corrosion development is known and assessed by experts. For Bayesian inference on the corrosion rate, we refer to Kallen & van Noortwijk (2005).

A defect (corroded spot) is said to be unacceptable if its depth exceeds a corrosion allowance depth, say y. Suppose that a defect is present on a particular location at time zero and it grows according to a gamma process. The first time at which the corrosion allowance is consumed by the growing defect, denoted by T, has the following cumulative distribution function:

$$F_{T_y}(t) = P\{T_y \leq t\} = P\{X(t) \geq y\}$$

$$= \int_{y}^{\infty} ga(x | \alpha t, \beta) dx. \tag{5}$$

In the literature, the distribution F in Equation 5 is known as the first hitting time distribution (Karlin & Taylor 1981). The space-time duality in Equation 5 follows from monotonic behavior of the gamma process trajectories (Noortwijk & Klatter 1999).

Figure 2 shows a sample path of the gamma process with linear expectation and variance given by Equation 4. Figure 3 shows the corresponding first hitting time probability density function of the gamma process reaching the corrosion allowance depth $y = 5$ defined in Equation 5.

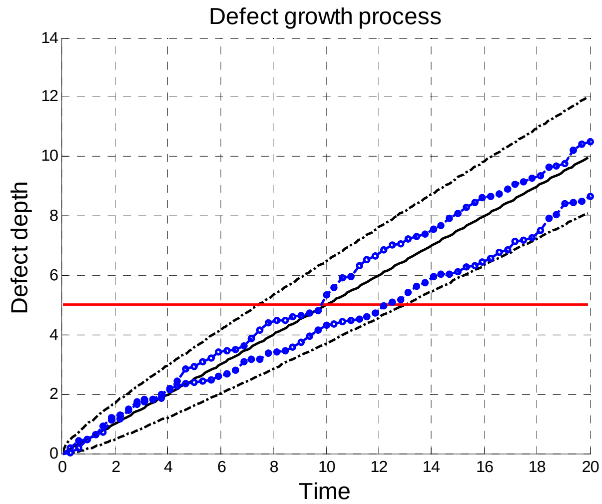

Figure 2: Two sample paths of a gamma process together with the mean, the 2.5% and 97.5% quantiles and the corrosion allowance depth $y = 5$.

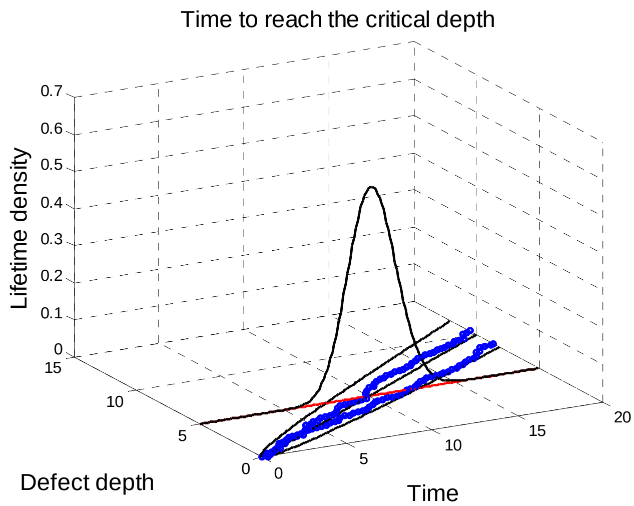

Figure 3: Hitting time density function of the gamma process reaching the corrosion allowance depth $y = 5$.

2.3 *Pitting/Localized corrosion process*

We define the pitting/localized corrosion process as a combination of the process of defect initiation and defect growth described in the subsections above. First, we assume that the defects initiate at event times from a NHPP and then they grow independently following the gamma process.

From the fact that the arrival times of the NHPP conditional on the number of arrivals can be treated as an order statistic from a sequence of *iid* random variables (Equations 2-3), it follows that the probability distribution function of the defect depth population at time t is given by:

$$P\{D_t < x\} = P\{X(t-S) < x | N(t) = 1\}$$

$$= \frac{1}{M(t)} \int_0^t P\{X(t-s) < x\} m(s) ds, \quad (6)$$

where $X(\cdot)$ is the gamma process.

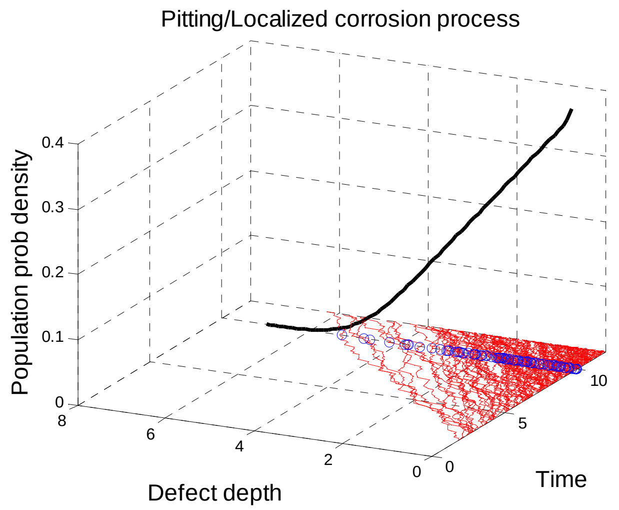

Figure 4: Sample paths of the pitting/localized process and defect depth population density function with $m(t) = qt^{q-1}$, $q = 2$.

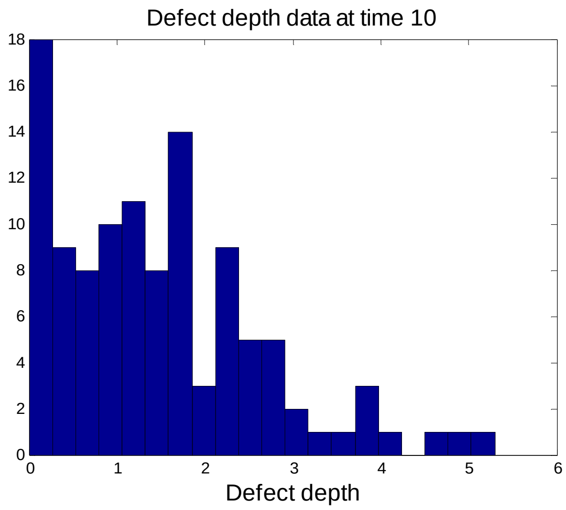

Figure 5: Defect depth histogram collected from the sample paths of the pitting/localized process shown in Figure 4 at time $t = 10$.

The probability in Equation 6 is the probability that a defect is smaller than depth $x > 0$ given that it has initiated up to time t according to a NHPP with intensity $m(t)$ and grown since the initiation according to the gamma process $X(\cdot)$.

Figure 4 shows sample paths of the localized corrosion process and the probability density function of the defect population at time $t = 10$, where the power law intensity function with the exponent $q = 2$ is used. Figure 5 shows the histogram of defect depths collected from the pitting/localized corrosion process at time $t = 10$.

3 SAMPLING INSPECTION

Sampling inspection is partial inspection of the system where only pre-selected sections are inspected. Let S denote the system surface (two dimensional plain: $S \subseteq \mathbb{R}^2$). We assume that the defects initiate in S according to the NHPP arrival times as described in Section 2.1. Suppose that an area A ($A \subseteq S$) is selected for inspection. It follows that the total number of defects in S can be written as:

$$N(t) := N(t;S) = N(t;A) + N(t;S \setminus A), \quad (7)$$

where the number of defects in A ($N(t; A)$) and the number of defects in $S \backslash A$ ($N(t; S \backslash A)$) are independent Poisson processes with the intensity function $p\lambda m(t)$ and $(1-p)\lambda m(t)$, respectively (Kulkarin 1995, page 219). This is called the Bernoulli splitting mechanism of the underlying Poisson process $N(t)$ with splitting parameter $0 < p < 1$ defined by:

$$p = \frac{E(N(t;A))}{E(N(t;S))}. \quad (8)$$

The parameter p represents the expected number of defects in the selected area A relative to the expected number of defects in the entire system S. In particular if defects are uniformly distributed in space then $p = |A|/|S|$ (i.e. the ratio of the size of the inspected area A to the size of the entire system S).

The resulting Poisson processes from the Bernoulli splitting mechanism are independent for a given value of the proportionality constant λ. However, if we assume that the proportionality constant λ from Equation 1 is unknown and it is represented by a random variable Λ, then by the total probability law and Bayes' theorem it follows that:

$$P\{N(t;S \setminus A) = n | N(t;A) = k\} =$$

$$= \int_\lambda P\{N(t;S \setminus A) = n | \lambda\} f_\Lambda(\lambda | N(t;A) = k) d\lambda, \quad (9)$$

where $f_\Lambda(\lambda)$ is the probability density function of Λ. Equation 9 demonstrates a Bayesian way to incorporate the information about λ from the inspected part of the system to make inferences about the non-inspected part. The probability in Equation 9 is the probability of having n defects in the non-inspected part of the system $S \backslash A$ given there are k defects present in the inspected area A. The distribution of Λ is updated from the prior to the posterior with observations from the inspected area A which is shown in Equation 21.

4 EVALUATION OF FAILURE PROBABILITY

System failure is defined as the event that '*in* (0, *t*] *at least one defect occurs that is deeper than the corrosion allowance depth y*'. In general, we can consider the process $N_x(t)$ counting the number of defects deeper than an arbitrary depth x up to time t and represent the failure probability in terms of the process $N_y(t)$ in the following way: $1 - P\{N_y(t) = 0\}$. In Section 4.1 we calculate the probability distribution function of the process $N_x(t)$.

4.1 *Exceedance probability*

In order to calculate the probability distribution of the process $N_x(t)$ we first compute the conditional probability of the number of defects deeper than x up to time t given n initiations. Using the fact that the event times from the NHPP conditional on the number of events can be represented by a sample of n *iid* random variables with the probability distribution given by Equation 3 and, independently of the initiation time, the probability that a single defect is deeper than x given it has initiated up to time t is $p_t = P\{D_t > x\}$ (Equation 6), it follows that the probability distribution of exactly k defects deeper than x given n defect initiations ($n \geq k$) is the binomial distribution given by:

$$P\{N_x(t) = k | N(t) = n\} = \binom{n}{k} p_t^k (1 - p_t)^{n-k}. \qquad (10)$$

Taking into account all possible numbers of defect initiations $N(t) = n \geq k$ it follows that:

$$P\{N_x(t) = k\} = \sum_{n=k}^{\infty} \binom{n}{k} p_t^k (1 - p_t)^{n-k} \frac{[\lambda M(t)]^n}{n!} e^{-\lambda M(t)}$$

$$= \frac{[\lambda p_t M(t)]^k}{k!} e^{-\lambda p_t M(t)}. \qquad (11)$$

Equation 11 implies that the process $N_x(t)$ is a NHPP with the expected number of defects exceeding x up to time t given by:

$$E(N_x(t)|\lambda) = \lambda p_t M(t) = \lambda M_x(t). \qquad (12)$$

The function $M_x(t)$ in terms of the first hitting time distribution F (Equation 5) is the following:

$$M_x(t) = \int_0^t F_{T_x}(t - s) m(s) ds. \qquad (13)$$

The failure probability as a function of t and fixed value of x using the Poisson process $N_x(t)$ is:

$$P\{\text{at least one defect deeper than } x \text{ within } (0,t]\} =$$

$$= 1 - P\{N_x(t) = 0\} = 1 - \exp\{-\lambda M_x(t)\}. \qquad (14)$$

Alternatively, we can use the Poisson process $N_x(t)$ as a function of $x > 0$ and derive an expression for the probability distribution function of the maximum defect depth up to time t (Max(t)). It follows that:

$$P\{\text{Max}(t) \leq x\} = P\{N_x(t) = 0\}. \qquad (15)$$

Similarly to the process $N_x(t)$ we can show that the process $N_x((r, t])$ which counts the number of defects that have initiated in the time interval $(r, t]$ and grown deeper than x up to time t, is a NHPP with the expected value $E(N_x((r, t])| \lambda) = \lambda M_x((r, t])$, where:

$$M_x((r,t]) := \int_r^t F_{T_x}(t - s) m(s) ds. \qquad (16)$$

4.2 *Prior distribution of the number of defects*

The proportionality constant of the NHPP intensity function in Equation 1 is assumed to be uncertain and modeled by a gamma-distributed random variable Λ (taking values $\lambda > 0$) with shape parameter $v > 0$ and scale parameter $a > 0$. The uncertainty in Λ expresses the uncertainty in the expected value and variance of the number of defects initiation $N(t)$.

The main reason for choosing the gamma distribution is that the gamma distribution is conjugated with respect to the Poisson likelihood function. This means that the posterior distribution of Λ is also a gamma distribution except that the parameters of the updated distribution are adjusted with the observations. Another reason for using the gamma distribution is that the gamma-Poisson mixture has a closed form known as the negative binomial distribution. This is useful since the number of defect initiations and the number of defects exceeding x (Equation 11) are both Poisson processes. This means that both the prior and posterior predictive probability of either the number of defect initiations or the number of defects deeper than x belong to the negative binomial family of distributions.

The probability of n defects up to time t integrated over the uncertain Λ has a negative binomial distribution given by:

$$P\{N(t) = n\} = \int_0^{\infty} P\{N(t) = n | \lambda\} ga(\lambda | v, a) d\lambda$$

$$= \frac{\Gamma(v+n)}{\Gamma(n+1)\Gamma(v)} \left(\frac{a}{a + M(t)}\right)^v \left(\frac{M(t)}{a + M(t)}\right)^n, \qquad (17)$$

where $n = 0, 1, 2, \ldots$

The expected value and variance of the unconditional number of defect initiations (the negative binomial distribution) are:

$$E(N(t)) = \frac{v}{a} M(t) = E(\Lambda) M(t), \qquad (18)$$

$$Var\left(N(t)\right)=\frac{v}{a^2}M(t)\left(a+M(t)\right)$$

$$=E(\Lambda)M(t)+Var(\Lambda)M(t)^2. \quad (19)$$

Note that if λ is known with certainty then $E(\Lambda)=\lambda$, $Var(\Lambda)=0$ and $N(t)$ has a Poisson distribution with known expected value and variance (i.e. $E(N(t))=Var(N(t))=\lambda M(t)$). However, if λ is uncertain then $(N(t)|\lambda)$ has a different Poisson distribution for different value of λ (see Figure 1). The parameters v and a of the gamma-distributed random variable Λ can be assessed by specifying the expected value and variance of the unconditional number of defects in time interval $(0,t]$, denoted by $\mu_{N(t)}$ and $\sigma^2_{N(t)}$ (Equations 18-19). It follows that if $\sigma^2_{N(t)} > \mu_t$ then the parameters v and a are given by:

$$v=\frac{\mu^2_{N(t)}}{\sigma^2_{N(t)}-\mu_{N(t)}},\ a=\frac{\mu_{N(t)}M(t)}{\sigma^2_{N(t)}-\mu_{N(t)}}. \quad (20)$$

Consider the example from Figure 1 where two Poisson processes are shown with $\lambda=1$, $\lambda=0.4$ and $M(t)=t^2$. If one believes that the true distribution of the number of defect initiations lays between the situation with $\lambda=1$, where the number of defects roughly varies between 80 and 120, and the situation with $\lambda=0.4$, where the number of defects roughly varies between 27 and 53, then a suitable choice for the negative binomial distribution would be the one presented in Figure 6 ($\mu_{N(t)}=70$, $\sigma^2_{N(t)}=760$) for which the number of defects varies between 26 and 133 (2.5% and 97.5% quantiles from the negative binomial distribution).

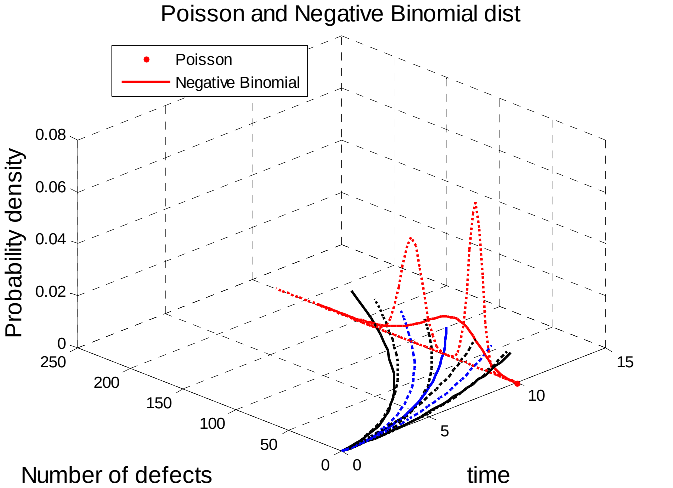

Figure 6: Number of defects distribution at time $t=10$: Poisson for fixed $\lambda=0.4$ ($\mu_{N(t)}=\sigma^2_{N(t)}=40$), $\lambda=1$ ($\mu_{N(t)}=\sigma^2_{N(t)}=100$) and Negative Binomial ($\mu_{N(t)}=70$, $\sigma^2_{N(t)}=760$) with 2.5% and 97.5% quantiles, $M(t)=t^2$.

4.3 *Prior and Posterior predictive failure probability given sampling inspection*

As mentioned before the gamma-Poisson mixture in Equation 17 is a negative binomial distribution. Since the failure probability (Equation 14) can be represented by the occurrence probability of the number Poisson events $N_x(t)$, it follows that the prior failure probability of no defects exceeding x in the entire system S is given by:

$$P\{no\ defects\ deeper\ than\ x\ within\ (0,t]\ in\ S\}=$$

$$=P\{N_x(t)=0\}=\left(\frac{a}{a+M_x(t)}\right)^v, \quad (21)$$

where $M_x(t)$ is defined in Equation 13 and v and a are the parameters of the prior gamma distribution of Λ. In general the probability of n defects exceeding x is given by Equation 17 where $M(t)$ is replaced by $M_x(t)$.

In order to derive the posterior predictive failure probability, we first determine the posterior distribution of Λ. As mentioned in Section 4.2 the posterior distribution of Λ given the Poisson number of defect initiations found in the inspected area A at time t with splitting parameter p is again a gamma distribution given by:

$$f_\Lambda\left(\lambda|N(t;A)=k\right)=ga\left(\lambda|v+k,a+pM(t)\right), \quad (22)$$

where k is the number of defects observed in the area A. Now, using Equation 17, the Poisson process $N_x(t)$ and the posterior distribution of Λ (Equation 22), it follows that the posterior probability of no defects exceeding x in S at the inspection time t (which depends only on the uncertainty in the non-inspected part of the system $S\backslash A$) is:

$$P\{N_x(t;S\setminus A)=0|N(t;A)=k\}$$

$$=\left(\frac{a+pM(t)}{a+pM(t)+(1-p)M_x(t)}\right)^{v+k}. \quad (23)$$

Finally, the probability of no defects deeper than x in S within time interval $(0,t]$ given the inspection of A at time $r<t$, where k defects with depths $d_1,\ldots,d_k$ were observed each smaller than x is:

$$P\{no\ defects\ deeper\ than\ x\ within\ (0,t]\ in\ S|$$

$$|N(r;A)=k,\max_{i=1,\ldots,k}(d_i)<x\}=$$

$$=P\{N_x(t;S)=0|N(r;A)=k,\max_{i=1,\ldots,k}(d_i)<x\}$$

$$=P\{N_x(t;S\setminus A)=0,N_x((r,t];A)=0|N(r;A)=k\}$$

$$\cdot P\{\max_{i=1,\ldots,k}(d_i+X(t-r))<x\}$$

$$= \left(\frac{a + pM(r)}{a + pM(r) + pM_x((r,t]) + (1-p)M_x(t)} \right)^{v+k}$$

$$\cdot \prod_{i=1}^{k} \left(1 - F(t - r \mid x - d_i)\right), \qquad (24)$$

where $N_x((r, t])$ is a Poisson process defined in Section 4.1 and $M_x((r, t])$ is defined in Equation 16.

5 EXAMPLE

For illustration purposes we present an example, where an inspection of the system is carried out with 30%, 60%, 90% and 100% coverage at time $t = 10$ years (the time since installation). The purpose of inspection is to verify the number of defects in the entire system using sampling inspection. This is achieved based on updating the probability distribution of the proportionality constant λ with the observations from the sampled inspection areas and incorporating this information to the entire system. The information about the rate of defect growth and the defect initiation in time ($M(t)$) is not intended to be updated with the inspection results. It is assumed that this information is incorporated in the model from other sources (e.g. defects growth monitoring, laboratory testing, expert assessment). For illustration purpose only, we assume perfect inspection which means that there is neither measurement error nor imperfect detection in the performance of inspection. In principle, however, a detection threshold can be defined above which all defects are recordable and this can easily be incorporated in the model.

The system is assumed to suffer from defects uniformly distributed in space what implies that the coverage is $p = |A|/|S|$ (Equation 8). We assume that the defects initiate according to the power law NHPP with exponent $q = 2$ ($M(t) = t^2$). Based on the assessment of the possible number of defects at time $t = 10$ ($\mu_{N(t)} = 250$, $\sigma^2_{N(t)} = 63000$) resulting in no more than 925 defects with probability 0.025 and no less than 5 defects with probability 0.025 the parameters v and a of the gamma distributed Λ are calculated using Equation 20. Regarding the wall loss process we assume a stationary gamma process with linear shape function as shown in Equation 4 with $\alpha = 5$ and $\beta = 10$. This means that the annual wall loss for a corrosive location in the system ranges between 0.16 and 1.02 units of wall thickness (2.5% and 97.5% quantile from the gamma distribution) with mean 0.5 units. The corrosion allowance depth is set to be $y = 8$, which means that in approximately 10.87 years we would expect a defect which initiated at time 0 crossing the barrier y with probability 0.001.

A random sample from the deterioration process is drawn resulting in 141 defects up to time $t = 10$. Inspection with 30% coverage results in 47 defects, 60% in 77, 90% in 122 and 100% in 141. In this example, the true maximum defect depth ($d_{max} = 4.57$) is only found in the 100% inspection what is shown in Figures 7-8.

Figure 7 shows the maximum defect depth probability density function at the inspection time $t = 10$ in the non-inspected part of the system. It can be seen that the densities corresponding to bigger coverage are shifted to the left. This is expected since the remaining number of defects in the non-inspected part of the system gets smaller resulting in smaller possible extremes. However, the spread of the densities (the variance of the maximum defect depth in the non-inspected part of the system) increases as the coverage increases indicating that we may have just one but deep defect outside the inspection areas. For instance, in the present example the true maximum was only found in the 100%-coverage inspection. The true maximum defect was not detected even in the 90%-coverage inspection, because it occurred in the remaining 10% of the system surface.

Figure 8 shows the prediction of the maximum defect depth in the entire system two years after inspection ($t = 12$) for sampling inspection. These densities are calculated using the probability in Equation 24.

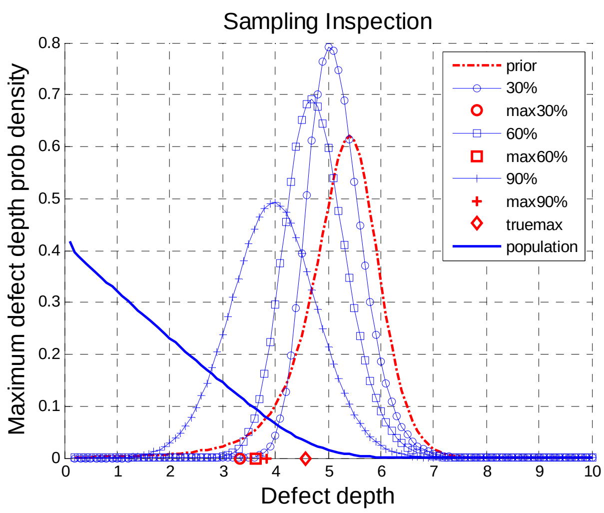

Figure 7: Maximum defect depth probability density function at the inspection time $t = 10$ in the non-inspected part of the system for sampling inspection (Equation 23). The population probability density function (Equation 6). The prior maximum defect depth probability density function in the entire system (Equation 21). The *truemax* is the true maximum and the 'max 30%', 'max 60%' and 'max 90%' correspond to the observed maximum for the inspection with 30%, 60% and 90% coverage, respectively. Referred equations show the formulas for the cumulative distribution function.

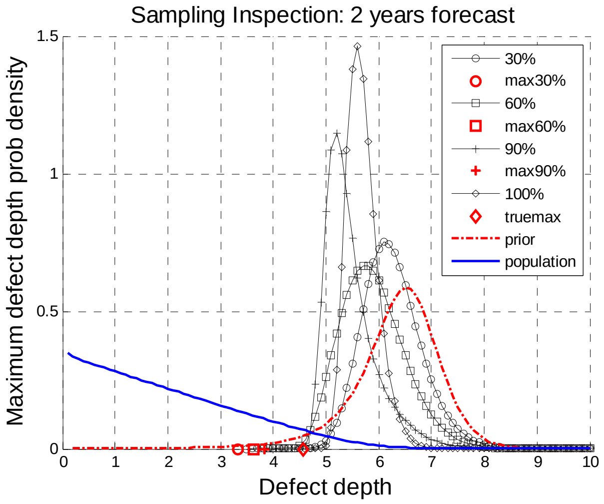

Figure 8: Probability density function of the maximum defect depth in the entire system *S* two years after inspection for sampling inspection (Equation 24). The population probability density function t = 12 (Equation 6). The prior maximum defect depth probability density function in the entire system at time t = 12 (Equation 21). Referred equations show the formulas for the cumulative distribution function.

6 CONCLUSIONS

We have presented a method to model the deterioration process of steel structures suffering localized corrosion damage. The deterioration process is a combination of two stochastic processes, namely the process of defect initiation and the process of defect growth. We calculate the time-dependent reliability of a deteriorating structure based on the assumption of a non-homogeneous Poisson process (NHPP) with intensity having an uncertain proportionality constant for the number of defect initiations and a stationary gamma process for the defect-depth growth (wall penetration). The probability that in the operational time no defect occurs that is deeper than the critical depth (corrosion allowance) is calculated a priori. Given the observed number of defects in the inspected part of the system obtained by sampling inspection, the posterior predictive failure probability is derived.

We show that the failure probability as well as the maximum defect depth probability distribution (most frequently desirable estimate in modeling extreme corrosion damage) can be represented in terms of the Poisson process $N_x(t)$ which models the number of defects that have initiated and exceeded depth x up to time t. This representation allows incorporating information about the defect morphology which includes the defect growth and the shape of the expected number of defect initiations in time that may come from other sources than inspection.

Additionally, given a detection threshold h of an imperfect detection device is specified, one can consider the observations above threshold h by replacing the Poisson process $N(t)$ of the defect initiations with the Poisson process $N_h(t)$ of defects above threshold h.

Another advantage of the presented model is that the failure probabilities are all related to the negative binomial distribution, which makes the model attractive from the computational point of view.

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